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Defining strategies to win in the Internet market

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Abstract

This paper analyzes a model for the competition dynamics of web sites in the Internet, based on the Lotka–Volterra competition equations. This model shows the well known appearance of a winner-take-all characteristic and is based in the nonvalidity of traditional offer and demand equilibrium theory of these kinds of markets. From the stability analysis of the model, we establish a series of rules which are useful for defining strategies in the Internet market. One of the most important results that emerge from this simple model is the appearance of some unexpected phenomena related to the collaboration and competition between sites. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

One of the most important drawbacks when planning the development of web sites is the absence of realistic mathematical models of the Internet markets. In the same way in which we can find models for the different phenomena in the traditional economy (equilibrium models, offer and demand models, competition models, etc.), it could be useful to develop mathematical explanations for the Internet business. In the past few years, some models of the competition dynamics of the Internet, and other phenomena related to the World Wide Web, have emerged [1–3]. Precisely, in Ref. [1], a model based on the Lotka–Volterra competition equations for n variables was first used

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considering complete symmetric conditions for the competition parameters. In this paper, we are using the same model for the case of three variables, since it is the simplest to account at the same time for collaboration and competition. The model does not consider any random effects, which could be desirable for a more real description. However, we want to point out that our main interest resides in the competitive dynamics of the model. Furthermore, we believe that this is a good starting point to gain a better understanding of the dynamics of the web sites. Seasonal effects, which might be modeled by external periodic perturbations or random perturbations, to take into account fluctuations on the system or the unpredictable behavior of the Internet users, could be useful in further developments. Stochastic effects for a generalized Lotka–Volterra model have been included in Ref. [4].

One of the main objectives here is to show how unexpected behaviors of these markets are predicted by simple models. Results from our analysis make it possible to obtain some rules, which can be useful to understand the competitive dynamics between the different web sites. As a matter of fact, one of the conclusions of our work is that very new and interesting phenomena emerge from the models when cooperation between sites is considered. In the same way in which some kind of ants and mushrooms cooperate in particular ecosystems complementing its capabilities and surviving like a single being, little web sites can collaborate with each other to avoid being destroyed by powerful Internet corporations. Even more interesting is the fact that the cooperation between sites drives to better economic results in terms of investment revenues. Numerical simulations show that the investment necessary for new companies to get into a particular Internet market segment is higher when one tries to accomplish it by developing a large single site. The cost can be extremely reduced by introducing little cooperating sites that complement its contents and services.

The organization of this paper is as follows: First, we describe the mathematical model and compute the fixed points and its stability. These results are used later to perform an analysis of the different kinds of markets. This previous analysis is used to define strategies for the web sites. Finally, a critical review of the model is performed and the conclusions are presented.

2. Description of the model

Every real-world market is complicated enough to be never fully explained by a mathematical model; nevertheless, we can create very simple models of market segments that present the main characteristics of the real Internet behavior. A complete model of the competition dynamics should take account of so many different effects and influences that would be impractical for simulation and analysis purposes. Nevertheless, in Ref. [1], it has been shown that the main characteristics of the Internet markets can be reproduced from a very simple model based on the Lotka–Volterra competition equations. This model can be described, in the general case of n different competitors, as a system of n nonlinear differential equations of the form,

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} = f_i\left(\alpha_i\beta_i - \alpha_i f_i - \sum_{i\neq j}\gamma_{ij}f_j\right), \tag{1}$$

where f_i is the fraction of the market which is a customer of site i, α_i is the growth rate which measures the capacity of site i to grow, β_i is the maximum capacity which is related to the saturation value of f_i (the maximum value f_i can reach) and γ_{ij} is the competition rate between sites i and j.

Strictly speaking, the model has been developed taking f_i as the fraction of the people being aware of the site i existence, but it is logical to think that this fraction must be strongly related to the number of visits to site i and thus with the number of customers. It is important to understand that a single user can be a client, at the same time, of all sites. Thus, it is possible to find market segments where $f_i = 1$ for all $i \in \{1,\ldots,n\}$(that would mean that 100% of the market population visit all sites, or is aware of all sites existence).

Parameter α_i is called the growth rate of site i, and higher values of α_i imply faster development of the sites within the market. In the Internet, sites having fast growth are those that offer very interesting contents, so we can relate this parameter to the quality of the contents of the web site. Sites having very interesting contents grow faster than sites having old-fashioned low quality contents. Due to that, we can define α_i as the *quality* of the contents of site i. We are not concerned about how to measure α_i in a given real web site, we just want to remark that, the higher the α_i, the higher the quality of the site.

Parameter β_i is the maximum capacity of site i. It determines the maximum fraction of the market that a given site can afford. The maximum capacity is related to the number of simultaneous connections a site may maintain, thus increasing the capacity is just a fact of having a more powerful server site. In order to simplify the model, we can imagine that sites can update their hardware and software fast enough to be never surprised by traffic congestion problems. From a dynamical point of view sites behave as having a maximum capacity of 1, what means $\beta_i = 1$ in all cases for our analysis.

Parameter γ_{ij} is called the competition rate between sites i and j. The competition rate measures the fraction of customers that site i looses because of site j. The stronger the competition rate between the two sites, the lower is the probability of finding customers being a client of both sites (being aware of the two sites existence) at the same time. The competition rate can be seen as a measure of the degree of incompatibility of the sites, which means that, if two sites are in strong competition, then customers may visit one or the other, but never both of them. This can be related to the likeness of the contents and the services offered by the sites, a customer rarely visits two sites having the same contents and offering the same services or products. For electronic shops the term services should also take account of prices, quality of products, etc. We are not concerned about how this parameter can be measured in a real web site, we just want to notice that γ has an influence on the site's growth, and that sites can modify its competition rate by modifying its contents, services, products, prices, etc. We assume the competition rate of two sites to be symmetric (i.e., $\gamma_{ij} = \gamma_{ji}$). Although this hypothesis is not necessarily verified in some particular kinds of markets, it is reasonable to think that under normal situations it may hold.

There are some kinds of markets, where the analysis of the model can be performed for the general case of n sites competitors; nevertheless, it is not feasible in other types of markets that we are going to study. Sometimes, it is going to be necessary to make a more restrictive assumption: to reduce the analysis of the competition dynamics to a market segment having only three competitors. This assumption, being indeed very restrictive, presents a lot of novel behaviors and characteristics and is easy enough to be treated analytically in the most interesting cases. This restriction can also be interpreted as modeling a market segment by taking only the three main competitors, considering that the rest do not interfere in the dynamics of the three most important ones. Thus, the model we use for the competitive dynamics of the market segment is given by

$$\begin{aligned}
\dot{f}_1 &= f_1(\alpha_1 - \alpha_1 f_1 - \gamma_{12} f_2 - \gamma_{13} f_3)\,, \\
\dot{f}_2 &= f_2(\alpha_2 - \alpha_2 f_2 - \gamma_{21} f_1 - \gamma_{23} f_3)\,, \\
\dot{f}_3 &= f_3(\alpha_3 - \alpha_3 f_3 - \gamma_{31} f_1 - \gamma_{32} f_2)\,.
\end{aligned} \tag{2}$$

As it was stressed before, this three-variable model is the simplest to account at the same time for collaboration and competition, and hence we use it for our study.

3. A classification of the markets

Depending on the parameter values of the model, we can define different kinds of markets:

- We say that site i is in strong competition with site j when $\gamma_{ij} > \alpha_i$.
- We say that site j collaborates with site i when $\gamma_{ij} < \alpha_i$. The model does not take into account any real collaboration phenomenon between sites, but under low competition rates, sites are able to evolve following their own dynamics and not being interfered by any other rival.
- A set of sites form an alliance when they are under collaboration conditions between each other, but under strong competition conditions with the rest of the rivals.
- A given market exhibits a winning site when there exists a site $i \in \{1, \ldots, n\}$ that verifies $f_i = 1$ and $f_j = 0$ for all $j \neq i$. In the same way, a given market presents a winning alliance when there exists an alliance that verifies $f_i > 0$ for all the sites belonging to the alliance and $f_i = 0$ for the rest.
- We say that a site *wins* in a market when it is a winning site or when it belongs to a winning alliance.

Our interest is especially concentrated on three particular kinds of markets that are as follows:

(1) *Completely collaborative markets*. In this kind of market, all competitors collaborate, they are under weak competition conditions. This kind of market allows all competitors to coexist maintaining each one a fraction of the market, that means that all sites win. The fraction of the market each site controls will depend, sometimes only on the characteristics of the market, and sometimes on the characteristics of the markets and on the initial condition of the sites.

(2) *Completely competitive markets*. In this kind of market, all the competitors are under strong competition conditions. It is known that these markets exhibit winner-take-all characteristics [5], that is, they have a winning site.

(3) *Mixed markets*. This kind of market presents the particularity that some competitors collaborate, but maintains, at the same time, strong competition conditions with the rest of the opponents. In this situation, the competitive dynamics may show diverse kind of behaviors. An exhaustive analysis of the stability of the fixed points of the equations is performed later in order to formally express the results. These markets may present a winning alliance.

Once we have introduced the different kinds of markets, one interesting question is how a site can get to win in a market. There are two main factors that determine this: first, the kind of market (i.e., the set of parameters that are involved into the equations) and second, the initial conditions of the competitors. The initial condition of a site i is the fraction of the market that is aware of the site i existence at the initial time (i.e., $f_i(0)$). All the past history of the market is represented in the initial conditions of its members. When a site starts offering its services in the instant $t=0$, the initial condition can be different from zero if the site has promoted its contents in a marketing campaign. The initial condition may also be nonzero when the site exists before the starting instant and already have a fraction of the market as a client.

4. Mathematical analysis of the model

For a better understanding of the model predictions, it is necessary to carry out an analysis of the fixed points and their stability. The general case of n competitors can be solved over the assumption that the growth rates and the competition rates of all sites are equal (i.e., $\alpha_i = \alpha$ and $\gamma_{ij} = \gamma$ for all i). Our goal here is to concentrate our efforts on the three competitors problem to find the particular situation, where an alliance of a pair of sites can win and make the third site disappear. This situation shows strong competition conditions between several sites but the market does not exhibit a winner-take-all characteristic. The eight different fixed points are:

- The trivial solution $P_0 = (0,0,0)$.
- Three fixed points P_1, P_2 and P_3 corresponding to the situation where the market presents a winning site. That is, one of the sites wins controlling the whole

market and the other competitors disappear. Thus, these fixed points are: $P_1=(1,0,0)$, $P_2=(0,1,0)$ and $P_3=(0,0,1)$.

- Three fixed points P_{12}, P_{13} and P_{23} that match the situation where an alliance of two sites wins making the third site disappear. These fixed points are:

$$P_{12}=\left(\frac{\alpha_1\alpha_2-\alpha_2\gamma_{12}}{\alpha_1\alpha_2-\gamma_{12}^2},\frac{\alpha_1\alpha_2-\alpha_1\gamma_{12}}{\alpha_1\alpha_2-\gamma_{12}^2},0\right),$$

$$P_{13}=\left(\frac{\alpha_1\alpha_3-\alpha_3\gamma_{13}}{\alpha_1\alpha_3-\gamma_{13}^2},0,\frac{\alpha_1\alpha_3-\alpha_1\gamma_{13}}{\alpha_1\alpha_3-\gamma_{13}^2}\right)$$

and

$$P_{23}=\left(0,\frac{\alpha_2\alpha_3-\alpha_3\gamma_{23}}{\alpha_2\alpha_3-\gamma_{23}^2},\frac{\alpha_2\alpha_3-\alpha_2\gamma_{23}}{\alpha_2\alpha_3-\gamma_{23}^2}\right).$$

Notice that the notation of the fixed points tells us the sites that win. It is important to remark that those fixed points make sense only in the nondegenerative case. For example, the solution for point P_{12}, will be meaningful only when $\alpha_1\alpha_2-\gamma_{12}^2\neq 0$. When this equation is zero, we have an infinite number of fixed points distributed over a straight line, where the alliance between sites 1 and 2 wins over site 3 making this last site disappear. Although this circumstance might occur, it is important to note that the fundamental properties of the fixed points remain constant.

- Last, we have the fixed point that corresponds to the situation where the three sites coexist $P_{123}=(z_1,z_2,z_3)$, where z_1,z_2,z_3 are the solutions of the following system of linear equations:

$$\begin{aligned}
\alpha_1 &= \alpha_1 z_1+\gamma_{12}z_2+\gamma_{13}z_3\,,\\
\alpha_2 &= \gamma_{21} z_1+\alpha_2 z_2+\gamma_{23}z_3\,,\\
\alpha_3 &= \gamma_{13} z_1+\gamma_{23}z_2+\alpha_3 z_3\,.
\end{aligned}\tag{3}$$

It can be noted that the fixed point is really a single point only when the three equations are linearly independent. If we have only two independent equations, we get a one-dimensional line of fixed points and if we have only one linearly independent equation, we obtain a plane of fixed points. However, the important fact is that all these fixed points have the characteristic of allowing the three competitors to survive controlling a fraction of the market segment.

To analyze the stability of the fixed points, we must compute the Jacobian matrix

$$\begin{pmatrix}
\alpha_1-2\alpha_1x_1-\gamma_{12}x_2-\gamma_{13}x_3 & -\gamma_{12}x_1 & -\gamma_{13}x_2\\
-\gamma_{21}x_2 & \alpha_2-2\alpha_2x_2-\gamma_{21}x_1-\gamma_{23}x_3 & -\gamma_{23}x_2\\
-\gamma_{31}x_3 & -\gamma_{32}x_3 & \alpha_3-2\alpha_3x_3-\gamma_{31}x_1-\gamma_{32}x_2
\end{pmatrix},\tag{4}$$

where x_1, x_2 and x_3 are the values of the fraction of the market controlled by each competitor. Concerning stability, if we calculate the eigenvalues of the Jacobian matrix for each fixed point we obtain the following:

- For the fixed point $P_0 = (0, 0, 0)$, the eigenvalues of the Jacobian are $\zeta_1 = \alpha_1, \zeta_2 = \alpha_2$ and $\zeta_3 = \alpha_3$. As all the eigenvalues are positive, this fixed point is always unstable.
- Regarding the three fixed points of the form P_i with $i = 1, 2, 3$, the following result is obtained. For $P_1 = (1, 0, 0)$ the eigenvalues of the Jacobian matrix are

$$\begin{aligned} \zeta_1 &= -\alpha_1 , \\ \zeta_2 &= \alpha_2 - \gamma_{21} , \\ \zeta_3 &= \alpha_3 - \gamma_{31} , \end{aligned} \tag{5}$$

so this fixed point is stable only when site 1 is in strong competition with sites 2 and 3 simultaneously. The conclusion we can extract is that the only way site 1 has to become a winning site of the market making the other disappear, is to compete strongly with the rest of the rivals. In a complete competitive market this fixed point is stable. It is important to notice that the parameter γ_{23} does not affect the stability of this fixed point, so it can also be stable in a mixed market situation, when sites 2 and 3 are allied. For the other two fixed points, the results are the same. Regarding $P_2 = (0, 1, 0)$ the eigenvalues are

$$\begin{aligned} \zeta_1 &= -\alpha_2 , \\ \zeta_2 &= \alpha_1 - \gamma_{12} , \\ \zeta_3 &= \alpha_3 - \gamma_{32} , \end{aligned} \tag{6}$$

and for $P_3 = (0, 0, 1)$ the eigenvalues take the following value:

$$\begin{aligned} \zeta_1 &= -\alpha_3 , \\ \zeta_2 &= \alpha_2 - \gamma_{23} , \\ \zeta_3 &= \alpha_1 - \gamma_{13} . \end{aligned} \tag{7}$$

- With reference to the three fixed points of the form P_{ij}, we take into consideration only the nondegenerative case, although these results could be generalized to cover all the possible circumstances. Fixed point P_{12} is going to be analyzed in detail, and this analysis can be easily generalized for the other two cases. We already know that

$$P_{12} = \left(\frac{\alpha_1\alpha_2 - \alpha_2\gamma_{12}}{\alpha_1\alpha_2 - \gamma_{12}^2}, \frac{\alpha_1\alpha_2 - \alpha_1\gamma_{12}}{\alpha_1\alpha_2 - \gamma_{12}^2}, 0 \right) .$$

The computation of the eigenvalues of the Jacobian matrix for this situation is impractical in the general case; however, we can make a number of light assumptions that make easier the resolution of the problem. We know that this fixed point is interesting only in case of having a two sites alliance between sites 1 and 2 against site 3. In such circumstances we could consider, in order to simplify the equations, that all sites have the same growth rate ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha$). Besides, as sites 1 and 2 are allied against site 3, they may present the same competition rate against their rival (i.e., $\gamma_{31} = \gamma_{32} = \gamma$). The parameter γ_{12} is independent of the rest. It measures the degree of alliance between sites 1 and 2. The lower the value of this parameter, the higher the grade of the alliance. Taking these assumptions, we can compute the eigenvalues of the Jacobian matrix to be

$$\zeta_1 = \frac{\alpha}{\alpha + \gamma_{12}}(\alpha + \gamma_{12} - 2\gamma)\,,$$

$$\zeta_{2,3} = \frac{\alpha}{\alpha + \gamma_{12}}(-\alpha \pm \gamma_{12}\mathrm{i})\,. \qquad (8)$$

Note that $\zeta_{2,3}$ form a pair of complex conjugate solutions, so we still have three eigenvalues for the Jacobian matrix. Although this analysis is not general, we can obtain an interesting conclusion from it; fixed points P_3 and P_{12} can coexist under particular competitive conditions. For example, take $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, $\gamma_{13} = \gamma_{23} = 2\alpha$ and $\gamma_{12} = \alpha/2$. In this particular case, due to Eqs. (5) and (8), we obtain that both fixed points, P_{12} and P_3 are stable. Nevertheless, the fixed point P_{12} is not compatible with a winner-take-all market. This proves that under asymmetric competition conditions the winner-take-all characteristic of the market disappears. Our numerical experiments show that this situation is maintained in markets having more than three competitors and with less restrictive competition conditions. Fixed points P_{13} and P_{23} are equivalent but with their respective constants.

- Regarding the fixed point P_{123} things are much more complex. It is not possible to compute the eigenvalues of the Jacobian matrix in the general case. In order to extract some information about the stability of the fixed points, it is necessary to make very strong and restrictive assumptions as complete symmetry in the competition and growth rates of the sites. For a complete analysis of this situation in a more general n sites competitors case, see Ref. [1]. The only important fact for us is to observe that, at least in the complete symmetric case, this fixed point is stable only under complete collaboration conditions and is unstable under strong competition conditions. Mixed conditions are not possible over the complete competition symmetric assumption, but the numerical results show that this fixed point may be never stable on mixed markets even with not so symmetric parameters. Mathematically speaking, for the three-competitor case, assuming complete symmetric conditions ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, $\gamma_{12} = \gamma_{21} = \gamma_{13} = \gamma_{31} = \gamma_{23} = \gamma_{32} = \gamma$), the fixed point can be expressed as:

$$P_{123} = \left(\frac{\alpha}{\alpha + 2\gamma}, \frac{\alpha}{\alpha + 2\gamma}, \frac{\alpha}{\alpha + 2\gamma}\right)$$

and we can affirm that P_{123} is stable if $\gamma < \alpha$ and unstable otherwise.

5. Analysis of the Internet markets

In this section, we attempt to analyze the different kinds of markets that the model includes for different choices of parameters. There are mainly three different kinds of markets, which will be described in the following.

5.1. Completely collaborative markets

This kind of market has the particular characteristic of presenting low competition rates for all sites. In order to simplify the analysis, we can consider a completely symmetric situation. This means that $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\gamma_{12} = \gamma_{21} = \gamma_{13} = \gamma_{31} = \gamma_{23} = \gamma_{32} = \gamma$. Fixed point P_0 is always unstable; in this situation, the fixed points P_1, P_2 and P_3 will also be unstable because the ζ_3 eigenvalue will be greater than zero, Eqs. (5)–(7). For the fixed points P_{12}, P_{13} and P_{23}, Eq. (8), the same situation is found because the eigenvalue ζ_1 will be greater than zero. So the only stable fixed point is P_{123} that has all eigenvalues lower than zero. So in a complete collaborative market, all competitors can coexist having each one a fraction of the market segment. The interesting fact here is that, for the nondegenerative case, the long term value of the fraction of the market controlled by each site is not dependent on the initial condition of the sites but only on the particular value of the competition rates and the growth rates. Thus, on such a market the initial marketing investment will not have an influence on the final portion of the market controlled by each site. We can see in Fig. 1, the time evolution of a market segment having three competitors where the market is completely collaborative. As the figure shows, all sites converge to control the same fraction of the market. Site 3 that starts with 60% of the population, finishes, in the long term, with the same profit than site 1 that begins only with 10% of the market.

5.2. Completely competitive markets

This particular kind of market can be only analyzed in complete symmetric conditions. Its main characteristic is the appearance of a winner-take-all characteristic; thus, the only stable fixed points are P_1, P_2 and P_3. The particular competitor that will become the winning site is determined by the initial conditions of the sites, the site having the highest initial condition is the one that gets the whole market population and the others disappear. So in this kind of markets under complete symmetric conditions, the main factor that determines the success of the site is the initial marketing investment. Under not so symmetric conditions, this rule does not hold any more, for example if the quality of the contents (the growth rate) of a site is better than those of the others, then the site can win and make the rest of competitors disappear, even if its initial marketing investments are weak. A typical complete competitive market situation with complete symmetry conditions is shown in Fig. 2, where the site having the highest initial condition wins and the rest disappear.

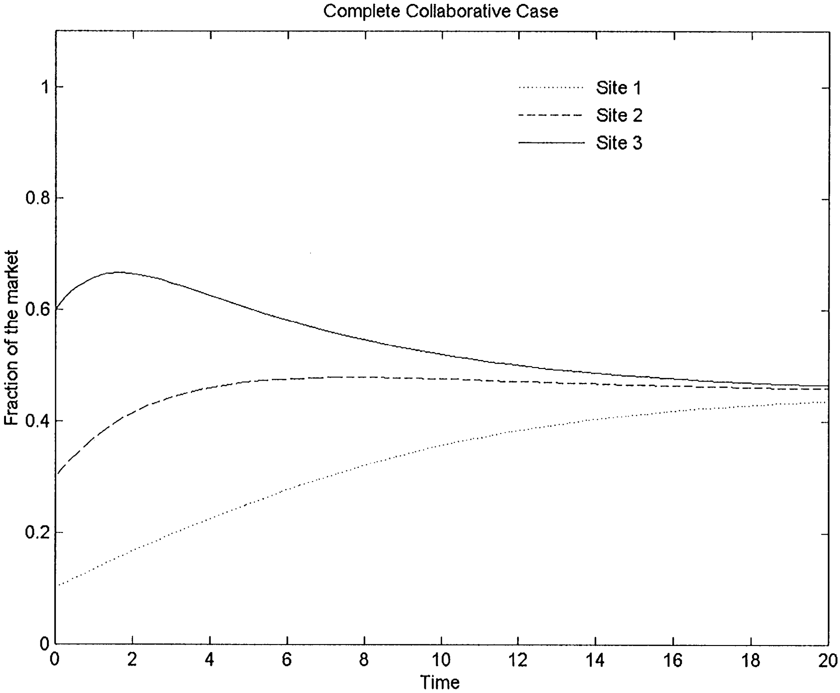

Fig. 1. Complete collaborative market, non degenerative case; All sites converge in the long term to the same fraction of the market. Parameters: $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\gamma_{ij} = 0.6$ (for all i and j). Initial conditions: $f_1(0) = 0.1$, $f_2(0) = 0.3$, $f_3(0) = 0.6$.

5.3. Mixed markets

As we said before, this is the most interesting kind of markets for our analysis. We know that all these markets have the particularity of exhibiting low competition rates between some sites and strong competition conditions for others. So in a mixed market there must exist, at least, one alliance between sites. Thus, in the simplified model of three competitors a mixed market must be composed of two sites collaborating against the third rival. For most cases in our discussions, we assume that sites 1 and 2 are allied against site 3. To simplify the analysis, we fix the equation parameters to the following values, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, $\gamma_{12} = \gamma_{21} < \alpha$, $\gamma_{13} = \gamma_{31} = \gamma_{23} = \gamma_{32} = \gamma > \alpha$. This implies that sites 1 and 2 have an alliance, sites 1 and 3 are under strong competition conditions and sites 2 and 3 are also under strong competition conditions. It can be proved using Eqs. (5)–(8) that the only stable fixed points in this situation are $P_3 = (0, 0, 1)$ and

$$P_{12} = \left(\frac{\alpha_1\alpha_2 - \alpha_2\gamma_{12}}{\alpha_1\alpha_2 - \gamma_{12}^2}, \frac{\alpha_1\alpha_2 - \alpha_1\gamma_{12}}{\alpha_1\alpha_2 - \gamma_{12}^2}, 0 \right) .$$

As can be seen, this market does not present any more a winner-take-all characteristic because if the fixed point P_{12} is stable, then the sites 1 and 2 can coexist at the same time, making site 3 disappear. The evolution depends on several parameters: the initial

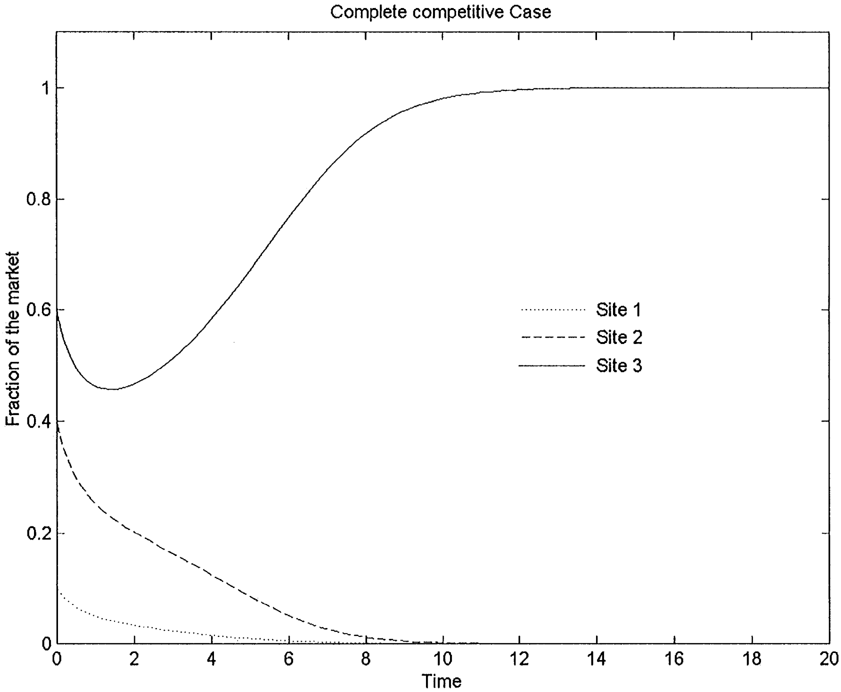

Fig. 2. Complete competitive case: The market presents a winner-take-all characteristic. Parameters: $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\gamma_{ij} = 2$ (for all i and j). Initial conditions: $f_1(0) = 0.1$, $f_2(0) = 0.4$, $f_3(0) = 0.6$.

conditions of the sites, the degree of alliance, the degree of competitiveness against the third rival and the growth rates of the sites. This set of parameters define the appropriate strategy to follow in order to make the market converge to the fixed point of our interest. An alliance of sites 1 and 2, destroying a more powerful rival that is site 3, is shown in Fig. 3. This phenomenon is produced by the collaboration of the two weaker sites. As can be seen in Fig. 4, using the same equations, but making the alliance to disappear (decreasing the degree of collaboration) site 3 wins, and the market presents again a winner-take-all characteristic.

6. Defining the strategy to win in the Internet markets

Now the problem is as follows. We have an Internet site, or we want to develop one, and it is necessary to define which strategy to follow in order to win. The objectives people look for when creating web sites are only known by themselves, but in a pragmatic point of view, we can think that people creating sites want to earn money. In the Internet, the money a site obtains depends strongly on the number of connections to the site. If the site is an e-commerce shop, we can consider that each person entering the site has a fixed probability p of buying something, so the bigger the number of

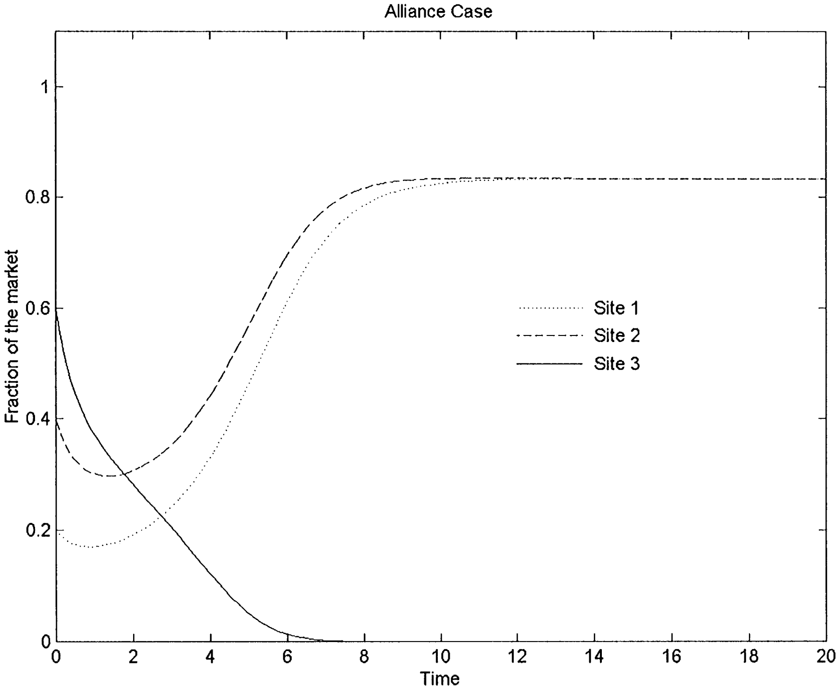

Fig. 3. Mixed market. The alliance of sites 1 and 2 beats site 3. Parameters: $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\gamma_{21} = \gamma_{12} = 0.2$, $\gamma_{13} = \gamma_{31} = \gamma_{23} = \gamma_{23} = 2$. Initial conditions: $f_1(0) = 0.2$, $f_2(0) = 0.4$, $f_3(0) = 0.6$.

connections, the bigger the number of customers. For a site living from marketing it is similar, that is, the money companies gives to the site for advertisements is proportional to the number of connections to the site. So we can consider that each time someone connects to the site, the site receives a quantity of money. Maximizing the money is just as simple as maximizing the number of connections the site receives. Due to this, we say that a site wins when it gets all the connections in the market segment and the rest receive none.

We can think of the initial condition just as the quantity of money necessary for making that fraction of the market being aware of the site existence. For already existing sites, this fraction of the market can be seen as the money a new incoming site should expend in order to get to this particular initial condition.

7. Strategies for the most powerful site

The most powerful site is the one having the highest initial condition. For example, imagine you are a site called power1.com, you want to define your strategy for the following years, the initial instant is today, the initial condition for you is the fraction of the market being aware of the power1.com existence today. Your degree of competition

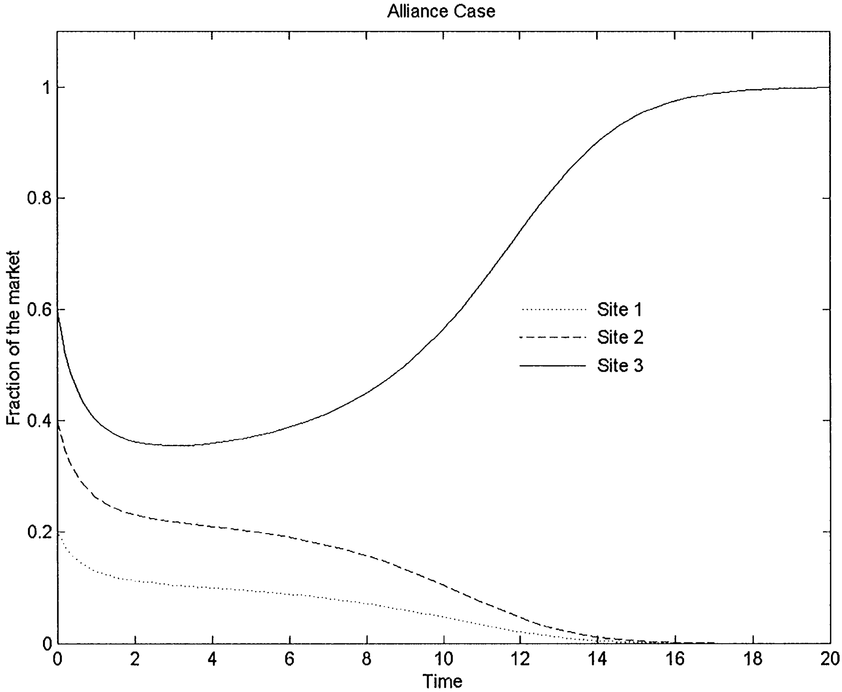

Fig. 4. The alliance between sites 1 and 2 has disappeared, site 3 wins. Parameters: $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\gamma_{21} = \gamma_{12} = 1.1$, $\gamma_{13} = \gamma_{31} = \gamma_{23} = \gamma_{23} = 2$. Initial conditions: $f_1(0) = 0.2$, $f_2(0) = 0.4$, $f_3(0) = 0.6$.

with the other sites and your growth rate are part of your strategy. In the following, we try to analyze the best type of market for the site power1.com.

7.1. Complete collaborative markets

Assume power1.com lives in a complete collaborative market and it decides also to collaborate. Using the general model for n sites competitors, with complete parameters symmetry, we can obtain that, the only stable fixed point is the one which makes all sites control a fraction of the market $f_i = \alpha/(\alpha + (n-1)\gamma)$. This means that, independent of the initial conditions of the sites all competitors will obtain the same profit. Even more, if a new site enters in the segment, all sites, including power1.com, will see how the profit decreases (observe that n is placed in the denominator). This situation is not right for power1.com because the incomings depend strongly on the others behavior and we cannot get to have the maximum possible profit the market offers (notice that the value of f_i in the equation is only 1 when $\gamma = 0$, so power1.com only gets to 100% of the market when the rest of the rivals do not compete). Besides, in the long term, all sites will get the same profit as ours, independent of the initial investments (the value of f_i is the same for all sites). So we can suspect that power1.com may be interested in orienting its strategy to have higher competition rates in the market.

7.2. Mixed markets

Mixed markets are harder to analyze. In order to clarify them, we can assume the three-competitor model in the market. Consider power1.com as being site 1 of the model, making an alliance with site 2 against site 3, where both are weaker than 1. The only stable fixed points, under complete symmetry conditions are $P_3 = (0, 0, 1)$, and

$$P_{12} = \left(\frac{\alpha_1\alpha_2 - \alpha_2\gamma_{12}}{\alpha_1\alpha_2 - \gamma_{12}^2}, \frac{\alpha_1\alpha_2 - \alpha_1\gamma_{12}}{\alpha_1\alpha_2 - \gamma_{12}^2}, 0 \right) .$$

Under these circumstances, it is very probable that the market will converge to the fixed point P_{12} (it would be difficult that site 3 beats the alliance of two more powerful rivals). This means that sites 1 and 2 will become equally powerful, besides the profit of site 1 is not necessarily the maximum predicted by the model (notice that in collaboration conditions $(\alpha_1\alpha_2 - \alpha_2\gamma_{12})/(\alpha_1\alpha_2 - \gamma_{12}^2) \leqslant 1$ because $\gamma_{12} < \alpha_2$) and it depends on the behavior of site 2. So this situation is not optimal for power1.com.

7.3. Complete competitive markets

Now suppose power1.com enters into a complete competitive market and decides also to compete strongly with the others. Under the assumption of n sites competitors, with complete symmetry in the parameters, we have n fixed points of the form $P_i(0, \dots 1, \dots 0)$. As we know this is a symptom of a winner-take-all market. In this case, the stable fixed point of convergence will be the one which makes the site with the highest initial condition to win and the others to disappear. For our case of interest, it means that power1.com wins controlling the whole market and the rivals disappear in the long term. Thus, it is obvious that the model predicts that the most powerful site is interested in making the market as competitive as possible.

8. Strategies for small web sites

When our site is not the most powerful of a market segment, the strategy must be defined examining the particular characteristics of the market. For each different type of market, an optimal strategy exists.

8.1. Complete collaborative markets

Imagine we have a site called small1.com that is not the strongest one, then we enter a market segment where all sites are in collaboration. We have two options, the first one is to compete with all of them and the second is to collaborate with all of them. The first will not be analyzed. The second option is the most interesting, in the case of an n sites market with complete symmetry conditions, we already know that the only stable fixed point is the one making all sites having $f_i = \alpha/(\alpha + (n-1)\gamma)$.

This means that we get the same market fraction as the rest of the competitors. The weaker the initial condition of our site with respect to the rest, the more interesting is this kind of market. Thus, this type of markets will be the one every little web site may look for, in order to be able to grow.

8.2. Complete competitive markets

We will face this situation when we are entering a market where all competitors are under strong competition conditions. The incoming site has only two possibilities, competition or collaboration. Competition implies its disappearance, and collaboration is addressed in the following section.

9. Making alliances

The situation we are to describe here is two small collaborating sites competing against a third powerful site. The challenging question is, what are the possibilities of the alliance winning site 3? Assuming the partial symmetric configuration, the only stable fixed points have been proven to be (Eqs. (7) and (8)): $P_3 = (0, 0, 1)$ and $P_{12} = (\alpha/(\alpha + \gamma_a),\ \alpha/(\alpha + \gamma_a), 0)$, what corresponds with the victory of the most powerful and of the alliance. In the general case, it cannot be calculated analytically which is the final fixed point of convergence, but our common sense and some numerical simulations allow us to decide the appropriate strategy. The most important ingredients determining the convergence are the initial conditions of the sites. It is practical to divide the problem into two subproblems. First, consider that the addition of the initial conditions of sites 1 and 2 is higher than that of site 3. This means that site 3 is the most powerful, but it is not as powerful as the strengths of the whole alliance put together. Second, we consider the situation where site 3 is even more powerful than sites 1 and 2 together and, in both cases interesting behaviors are shown by the model.

9.1. Site 1 plus site 2 is more powerful than site 3

This situation is found when the initial condition of site 1 plus the initial condition of site 2 is greater than the initial condition of site 3. In Fig. 5, we can see a series of pictures, where each pixel represents the result obtained by a web site after a certain integration time for a particular set of initial conditions and parameter values. A white pixel means the site has a fraction of the market of 1, a black pixel means the site has a fraction of the market of 0. A gray pixel represents a value between 0 and 1, the whiter the pixel the nearer the value of the market fraction to 1, the blacker the nearer to 0. The figure gives evidence that when collaboration is strong enough, the alliance wins.

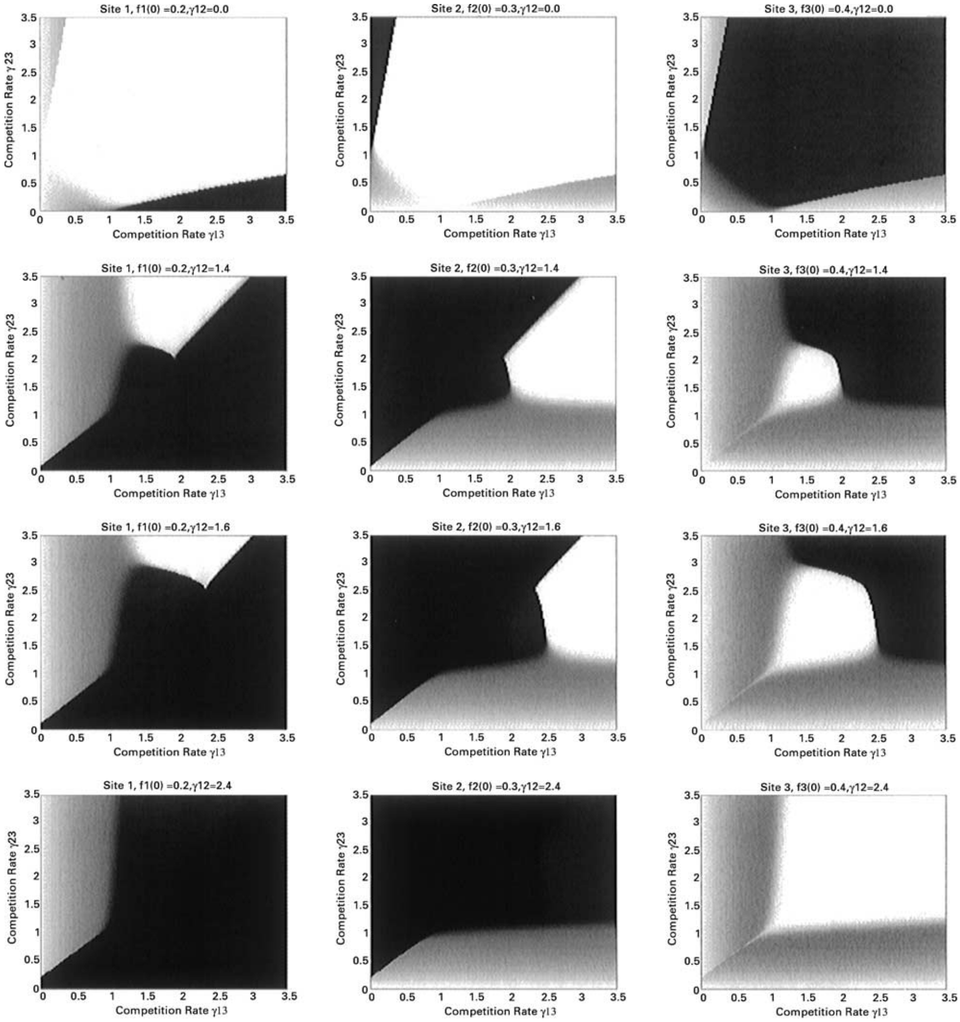

Fig. 5. Each picture pixel represents the fraction of the market, f_i, controlled by each site: black means 0 (the site has no visitors), white means 1 (all people in the segment market visit the site). Column 1 depicts the results for site 1, column 2 for site 2 and column 3 for site 3. The initial conditions of the sites for all pictures are $f_1(0)=0.2$, $f_2(0)=0.3$, $f_3(0)=0.4$. The x-axis represents different values of the competition rate between sites 1 and 3 and the y-axis represents those between sites 2 and 3. The upper row has been performed with a competition rate between 1 and 2 of $\gamma_{12}=0.0$, the second row with $\gamma_{12}=1.4$, the third row with $\gamma_{12}=1.6$ and the fourth row with $\gamma_{12}=2.4$. The integration time has a value of 30 for all pictures.

9.2. Site 1 plus site 2 is less powerful than site 3

This situation considers the case when the initial conditions of the alliance is smaller than the initial condition of 3. Our common sense says that it would be logical for site 3 to win under all circumstances in this situation, but this is far from the model

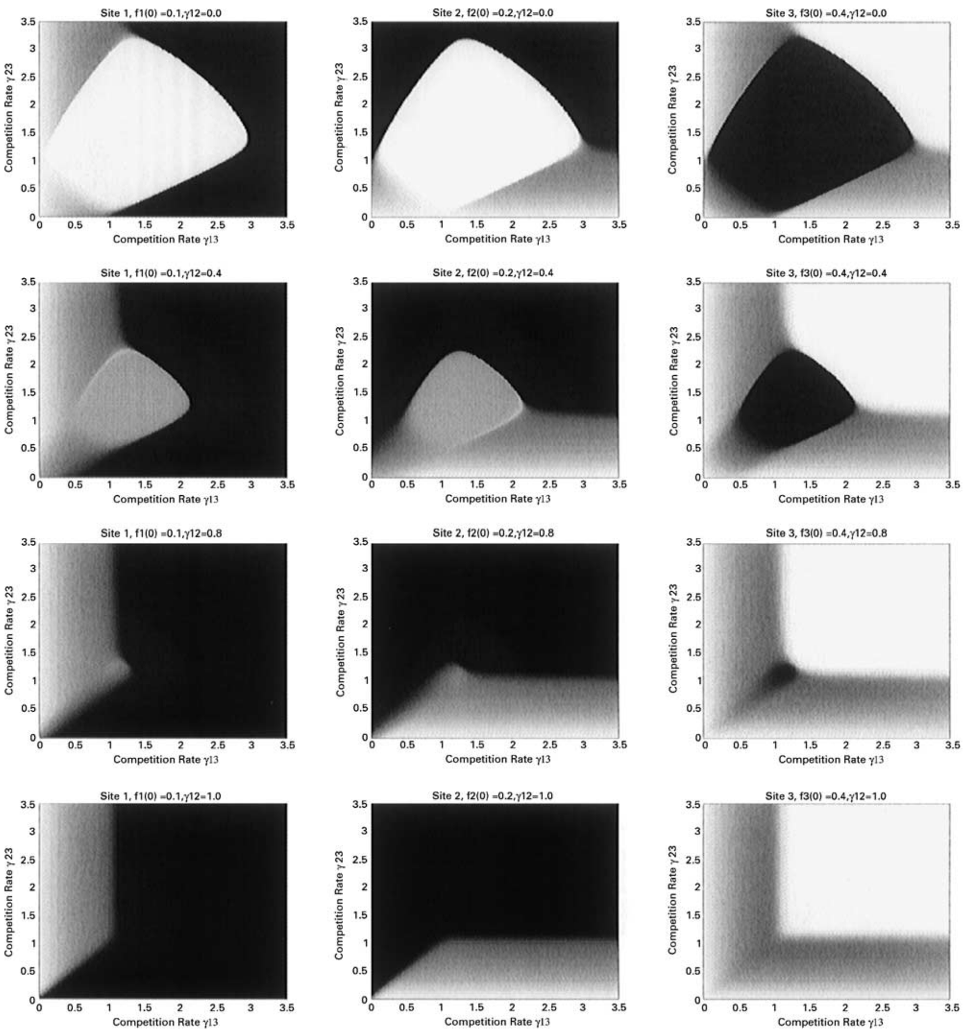

Fig. 6. Each picture pixel represents the fraction of the market, f_i, controlled by each site: black means 0 (the site has no visitors), white means 1 (all people in the segment market visit the site). Column 1 depicts the results for site 1, column 2 for site 2 and column 3 for site 3. The initial conditions of the sites for all pictures are $f_1(0) = 0.1$, $f_2(0) = 0.2$, $f_3(0) = 0.4$. The x-axis represents different values of the competition rate between sites 1 and 3 and the y-axis represents those between sites 2 and 3. The upper row has been performed with a competition rate between 1 and 2 of $\gamma_{12} = 0.0$, the second row with $\gamma_{12} = 0.4$, the third row with $\gamma_{12} = 0.8$ and the fourth row with $\gamma_{12} = 1.0$. The integration time has a value of 30 for all pictures.

predictions. The series of pictures shown in Fig. 6 exhibit an interesting behavior. When sites 1 and 2 are deeply allied and they compete not very hardly with site 3, then they have the possibility of winning, even if the sum of their initial conditions is under the one of site 3.

The series of pictures shown in Fig. 7 illustrate the behavior of the market for different initial conditions and competition rates. When the competition level between

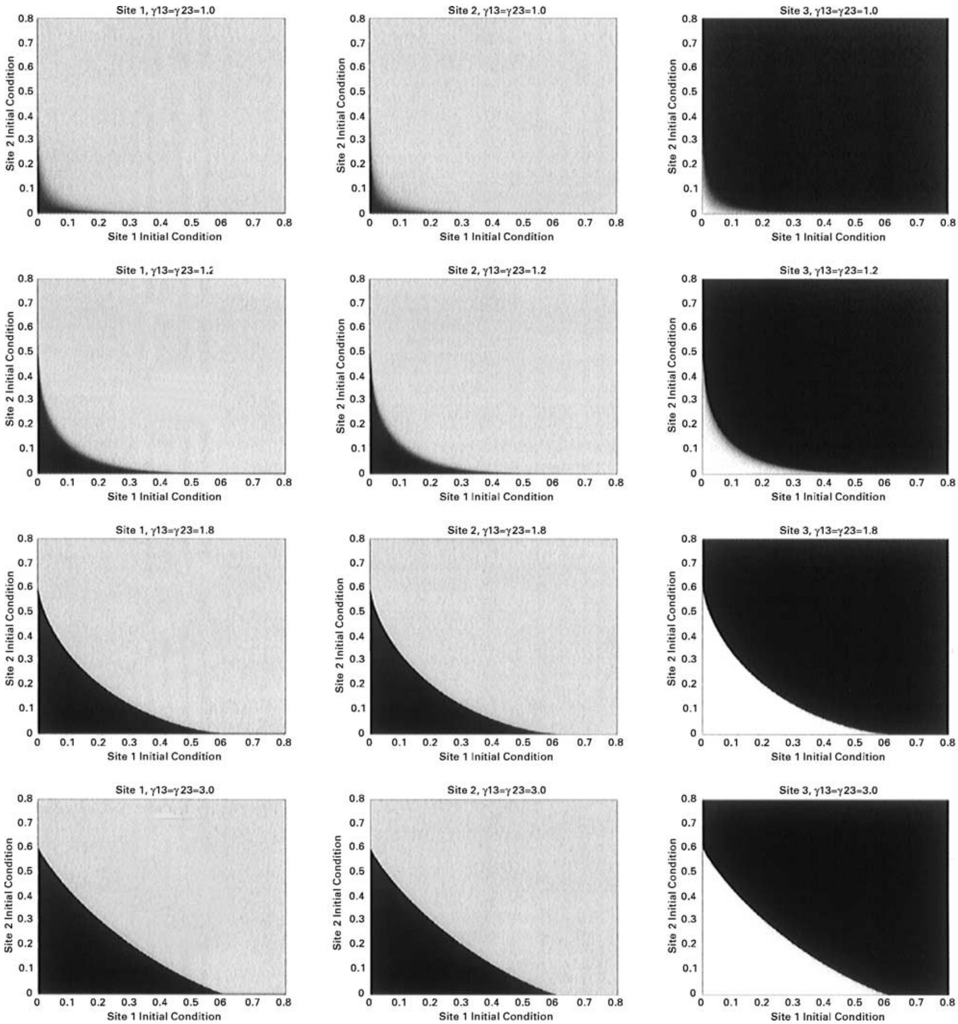

Fig. 7. Each picture pixel represents the fraction of the market, f_i, controlled by each site after an integration time: black means 0 (the site has no visitors), white means 1 (all people in the segment market visit the site). Column 1 depicts the results for site 1, column 2 for site 2 and column 3 for site 3. For all pictures, the following parameters are fixed: Competition rate between sites 1 and 2 $\gamma_{12} = 0.2$, initial condition of site 3 $f_3(0) = 0.6$. The x-axis represents different values of the initial condition of site $1 f_1(0)$ and the y-axis represents those of site 2 $f_2(0)$. The upper row has been performed with a competition rate $\gamma_{13} = \gamma_{23} = 1.0$, the second row with $\gamma_{13} = \gamma_{23} = 1.2$, the third row with $\gamma_{13} = \gamma_{23} = 1.8$ and the fourth row with $\gamma_{13} = \gamma_{23} = 3.0$. The integration time has a value of 30 for all pictures.

the alliance and the most powerful site is moderate, then the alliance can win, even though its initial conditions are much smaller than the ones of the most powerful site.

We want to stress here that this is one novel and interesting phenomenon emerging from the model. The rows are ordered presenting a degree of alliance from strong to weak. The interesting thing is that the alliance can beat site 3 only if the degree of collaboration is high and the degree of competition with site 3 is strong, but not too strong. In essence, this implies that in order to beat the powerful site, it will be cheaper to create two allied sites, than to create a single site with high initial condition. The development of the Internet supports this conclusion obtained from the model. It is known nowadays that new sites having a lot of success are thematic portals, that is, web portals dealing only with a particular and specific topic instead of trying to cover all kinds of information.

10. Advanced strategies

We have seen that the competition dynamics model that we are using works fine for the most simple examples, and seems to predict a number of phenomena that are intuitively true in the Internet markets. Nevertheless, the model also predicts the possibility of using strategies that are far from being intuitive, but that seem to work on the model, and possibly also in the Internet. We have called advanced strategies, all those singular strategies that can possibly be adapted to a real-world environment in order to design winning web sites.

An example of this can be described assuming a market segment in the Internet being controlled by two sites, sites 1 and 3. Suppose site 3 is the most powerful one, say it has 60% of the market, site 1 is not as powerful as site 3, it has just 40% of the market. Site 3, being the most powerful, is not interested in making alliances, since it knows in the long term it will finish by controlling the whole market, so it maintains a strong competition level, for example $\gamma_{13} = 2$, with site 1. Site 1 may be upset, it can do nothing except for investing money in marketing to get more than 60% of the market, but that is probably too expensive and cannot be done easily. Imagine that site 2 has just enough money to get 5% more of the market. Having 45% of the market would not solve their problems. Nevertheless, our model predicts that it can use an advanced strategy, the site can spend its money creating a new site and making an alliance with it. We are going to call site 2 this new site. Surely, site 3 is not worried about the presence of the small site 2, but the model predicts that the effect of this small site can destroy site 3. This situation is clearly shown in Fig. 8.

Another advanced strategy is based on the same principle, leaving a competitor to choose the competitive rate within a particular market. This particular situation is very rich and shows a large amount of different and interesting behaviors. One of the most surprising examples is the market where there are two really powerful sites, 1 and 2, competing hardly with each other. Site 1 has 60% of the market population and site 2 has 55%. In this context, we can imagine the third competitor site 3 entering the game with only 10% of the market. This new site is very small, in such a way that it might not receive any consideration by the other sites. Thus, the powerful sites 1 and 2 are destroyed by themselves, leaving site 3 to win. This situation is illustrated in Fig. 9.

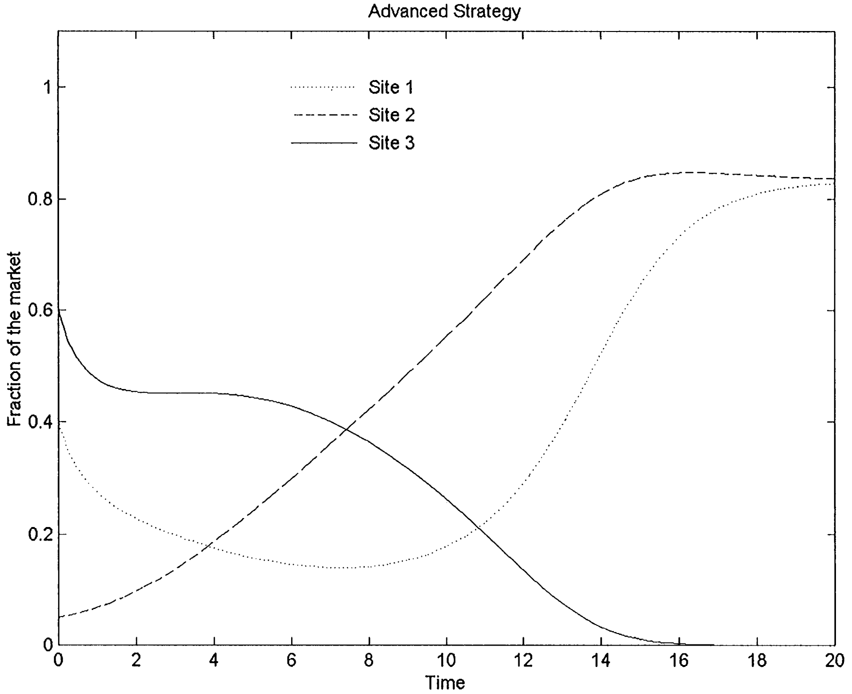

Fig. 8. Advanced strategy: The creation of little site 2 by site 1 beats powerful site 3. Parameters: $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\gamma_{21} = \gamma_{12} = 0.2$, $\gamma_{13} = \gamma_{31} = 2$, $\gamma_{23} = \gamma_{23} = 1.1$. Initial conditions: $f_1(0) = 0.4$, $f_2(0) = 0.05$, $f_3(0) = 0.6$.

11. Conclusions

We have seen that a very simple model predicts the main characteristics of the Internet markets and can be useful to define strategies, although the results should not be taken literally. There are a lot of different effects that the model does not contemplate. In particular, as we have mentioned in the paper, we are not considering random forces into the model. We have chosen the three-variable system, to show one of the most important aspects of our work, which is the nonlinear effects on the competition between web sites. Moreover, the model should also be able to evolve allowing sites to change their strategy dynamically. Nowadays, the parameters involved in the strategy, the competition rates essentially, are taken as constant values that do not change with time. It would be interesting to add the possibility of modifying these parameters depending on the market evolution; in this way, sites could change their strategies adapting them to the particular conditions of the market. Another drawback of the model is that the parameters remain too abstract: we are talking about the competition rates, the growth rates, the initial conditions and the evolution in time, but it is not really clear how these parameters can be interpreted in a real situation. An economist or a company leader would like to calculate the α and the γ of a particular

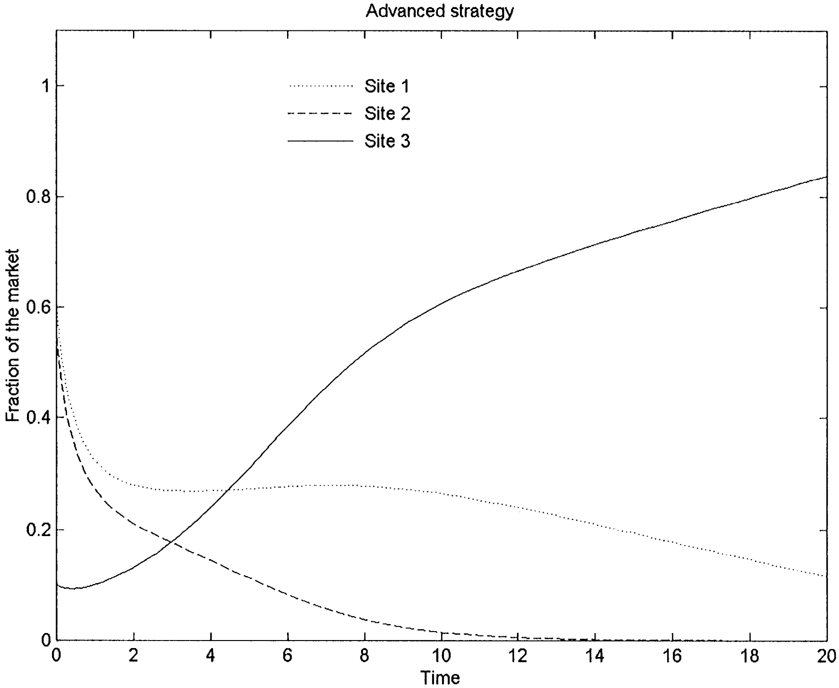

Fig. 9. Advanced strategy: Little site 3 beats powerful sites 1 and 2. Parameters: $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $\gamma_{21} = \gamma_{12} = 3$, $\gamma_{13} = \gamma_{31} = 1.2$, $\gamma_{23} = \gamma_{23} = 1.1$. Initial conditions: $f_1(0) = 0.6$, $f_2(0) = 0.55$, $f_3(0) = 0.1$.

web site. Although we know that their values are related to real characteristics of the sites like the interest of its contents and its similarity with other sites, this is not enough when we are trying to predict the evolution of a market segment through the model. Moreover, the time unit also remains too ambiguous; the model can predict an increase in traffic in, say, two units of time, but in order to deploy a realistic strategy it would be necessary to know if these two units mean 2 months or 2 years. Most of these indefinite items could be solved comparing the model results with traffic analysis statistics or real web sites. If we could find a market segment where the hypothesis of the model could be matched, assuming it is possible to estimate the value of the different parameters, we could easily decide what the appropriate unit of time is by simply comparing the predictions of the model with the real data. Unfortunately, Internet traffic statistics are not easy to obtain. Another possible improvement of the model is related to adding noise and seasonal effects to the model. This could take account of changes in the parameters due to external events. Internet users are less interested when they are on vacation than when they are at work, so the parameter α can be expressed as the sum of periodical forces modeling the seasonal, weekly and daily behaviors of the customers. Our first developments show that these effects can modify the predictions of the model and thus can induce modifications in the strategies. More advanced models should take account of complex phenomena like cross marketing (when a site has links to other sites), reinvestments of the profit, the presence of external investments, etc.

In spite of these imperfections, we believe that the model is a good starting point and an interesting tool to gain insight into the mechanism that govern the competition dynamics of the Internet markets. It shows that unlike the traditional material goods-based markets, the Internet is not driven by offer and demand principles, because the production cost of electronic goods does not depend on the number of replications. This makes that, once a site controls a market segment, the most powerful one, the rest of the competitors have little chance of finding a place in the segment, because the main one is able to fulfil all the market demand. It is also because of that, that a winner-take-all characteristic appears under strong competition conditions. Furthermore, the model gives a very important role to the alliances in the Internet. Until today, alliances of web sites have taken place in the form of one company absorbing another, but the electronic contents of two allied sites usually are far from being complementary. The model predicts that specializing the contents of allied web sites, dividing the contents according to the customers tastes, may give much better results than maintaining huge sites dealing with almost all possible themes a person can be interested in.

One could criticize that there is no solid evidence for the validity of the model. It is true that it would be necessary to perform an exhaustive analysis of real traffic statistics to verify it, but there are indications that prove that some of the behaviors predicted by the model are true [2]. On the other hand, the results obtained from the model seem to be in the same line than the conclusions obtained by analysts in the past few years, who predict the crisis of generic portals and propose thematic sites as the alternative for the Internet future.

As a summary, in this paper we have shown that the Lotka–Volterra competition equations can be an adequate model to describe the competitive dynamics of the Internet markets. We have seen how different sites may plan different strategies depending on the conditions of the particular market segment they are in: strong sites may look for high competition conditions, weak sites may look for noncompetitive markets or may ally with other sites if the markets are highly competitive. We also have seen how the traditional equilibrium theory is not applicable any more in this kind of markets. More complex models of the Internet dynamics, taking account of a larger number of phenomena, should be developed in order to create a more complete theory describing virtual markets.

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