Complex networks and the WWW market

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Abstract

This paper presents a model for the competition dynamics in the WWW market. We show that this problem is suitable to be analyzed in the framework of the theory of complex networks, representing each site by a vertex in a graph and each competitive interaction as an edge. Once the topology of the interaction network has been defined, we evaluate the dynamical evolution of the fraction of the market controlled by the sites through a set of differential equations based on the Lotka-Volterra equations. We show that, under these assumptions, some interesting and novel nonlinear effects emerge in this kind of markets.

Key words: Complex networks, competitive WWW markets, winner-take-all, collaborative networks.
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1 Introduction

Traditionally, scientists, and mainly physicists, have focused their efforts in the study and description of the constitutive components of the universe. The strategy most frequently used is to divide a particular problem until a concrete part of reality has been isolated. Then, it is analyzed separately, looking for the laws describing its individual behavior, and assuming that once we understand each part, it is easy to grasp the whole. Nevertheless, in the past years, a new branch of science with different principles has come to light. This new discipline proposes that the interaction between the pieces is, at least, as important as the individual behavior of the pieces itself. This new

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science has not a particular topic of research, it is a transdisciplinary subject covering different aspects of reality, which is called the science of complex networks.

Networks are everywhere in the real world. In the past few years there has been numerous publications in so different scientific and technological disciplines as: neurobiology [1], the metabolic pathways [2], the food webs in ecology [3], the World Wide Web (WWW) [4], the social interactions [5,6], etc. One of the most interesting applications of this tool is the description of economic relationships. The global economy is a network of national economies, which are themselves networks of markets, which are, in turn, networks of interacting producers and consumers.

Here we are particularly interested in some kind of complex networks appearing in a very particular field of economics, related to the competition dynamics in the WWW market. The structure of this paper is the following. First, we propose a simple dynamical model for the competition dynamics in the WWW markets, showing why the theory of complex networks is an appropriate framework for the analysis of these problems. Afterwards, by means of this model, we show that some interesting and counter-intuitive phenomenon appear on these markets due to the nonlinearity of the involved equations. Finally, some concluding remarks are extracted.

2 Competitive networks in the WWW

In the same way that we can find models for different phenomena in the traditional economy (equilibrium models, offer and demand models, competition models, etc.), it could be useful to develop mathematical explanations for the Internet business. Hence, our objective is to introduce a novel model for the competition dynamics on the WWW market. Similar models have been already introduced in the scientific literature in the past few years, mainly describing the competition dynamics in the Internet and other phenomena related to the World Wide Web [7]. The theory of complex networks emerges spontaneously as the most suitable framework to develop our model. The two basic ingredients of a complex network are present: a set of entities (the web sites) and an interconnecting topology describing their competitive structure. The individual evolution of the traffic on each of the sites is calculated following a model based on the Lotka-Volterra [8] competition equations.

In the general situation, we have n different competitors, so that the system must be described in terms of n nonlinear differential equations of the form,

$$\frac{df_i}{dt} = f_i(\alpha_i\beta_i - \alpha_i f_i - \sum_{i\neq j}\gamma_{ij}f_j), \quad (1)$$

where f_i is the fraction of the market which is a customer of site i, α_i is its growth rate, β_i is the maximum capacity, which is related with the saturation value of f_i (the maximum value that f_i can reach) and γ_{ij} is the competition rate between sites i and j.

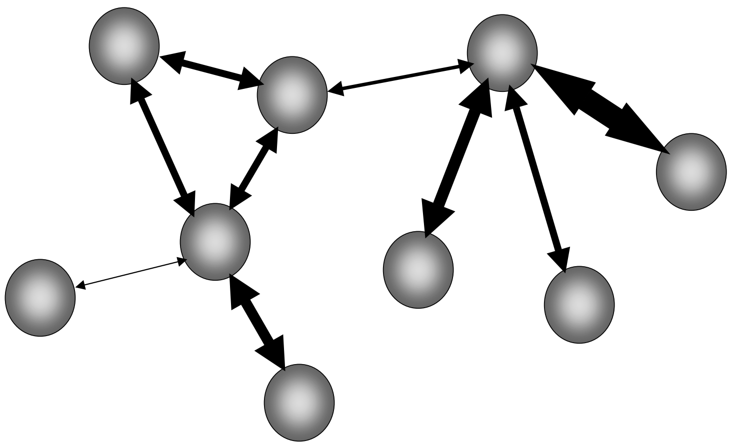

Fig. 1. This figure shows a network representing a market segment in the WWW. The vertices are sites and the edges are associated to their competitive interactions.

Each of these equations have $n+1$ terms. The first two of them describes the natural tendency of the population to grow ($\alpha_i\beta_i f_i$) and the competition of a site with itself ($\alpha_i f_i f_i$). The rest of the terms are associated to the competitive interaction of the site with the rest of the rivals ($-\sum_{i\neq j}\gamma_{ij}f_i f_j$). The interesting feature is that the set of γ_{ij} constants completely defines the interconnecting topology of a relational network. To understand why, just observe that it is possible to write the last terms of the differential equations using the following matrix-based expression

$$\begin{pmatrix} f_1 & 0 & ... & 0 \\ 0 & f_2 & ... & 0 \\ . & & & \\ 0 & 0 & ... & f_n \end{pmatrix} \begin{pmatrix} 0 & \gamma_{12} & ... & \gamma_{1n} \\ \gamma_{21} & 0 & ... & \gamma_{2n} \\ . & & & \\ \gamma_{n1} & \gamma_{n2} & ... & 0 \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ . \\ f_n \end{pmatrix} \quad (2)$$

where the matrix in the middle can be seen as the adjacency matrix of a directed weighted graph, and the weight of the connecting arrow between two nodes i and j is just γ_{ij}.

In these equations, higher values of α_i imply faster development of the sites within the market. In the Internet, sites having fast growth are those that offer very interesting contents, so we can relate this parameter to the quality of the contents of the web sites. The parameter called β_i is the maximum capacity of site i. In order to simplify the model, we can assume that sites behave as having a maximum capacity of 1. Finally, the parameter γ_{ij} is called the competition rate between sites i and j. The competition rate measures the fraction of customers that site i looses because of site j. This rate can be seen as a measure of the degree of incompatibility of the sites. That means that, if two sites are in strong competition, then customers may visit the first or the second one, but never both of them. Since, we assume that the competition rate of two sites is symmetric (i.e. $\gamma_{ij} = \gamma_{ji}$), a very important consequence of this assumption is that the adjacency matrix described in Eq.(2) becomes also symmetric. This means that the associated graph becomes undirected. In Fig. 1, a complex network representing this situation is shown.

3 Nonlinear effects in the collaboration between sites

Depending on the structure of the relational graph of the network, the analysis of the model, for the general case of n competitors, may be easy, difficult or infeasible. For example, when the considered topology is absolutely regular (all the γ_{ij} are equal), the graph is unweighted and we have a complete symmetric situation where the stability of the model can be fully evaluated analytically. Nevertheless, when a market with different competition rates is considered, the general case of n competitor is not tractable and it is necessary to adopt some restrictions making possible the evaluation of the stability. The mathematical details of this analysis can be found in [7]. Nevertheless, in this paper we are only interested in some of the aspects related to a set of interesting effects appearing in the competition dynamics.

Before describing them, it is necessary to make a couple of definitions to formally determine what we understand by collaboration and competition in this context. We say that site i is in competition with site j when $\alpha_i < \gamma_{ij}$. In the same way, we say that site i is in collaboration with site j when $\alpha_i > \gamma_{ij}$. In this circumstances, it is already known [8] that when all sites are in competition a *winner-take-all* characteristic emerges. This means that, in this kind of markets, there is only one competitor (the strongest one) who survives, making all the others to disappear. For this reason, small sites entering a new market segment must try to find a strategy to avoid this effect. What the model predicts [7] is that it is possible to create alliances between sites to break the *winner-take-all* characteristic by establishing mixed competition conditions where collaboration is present.

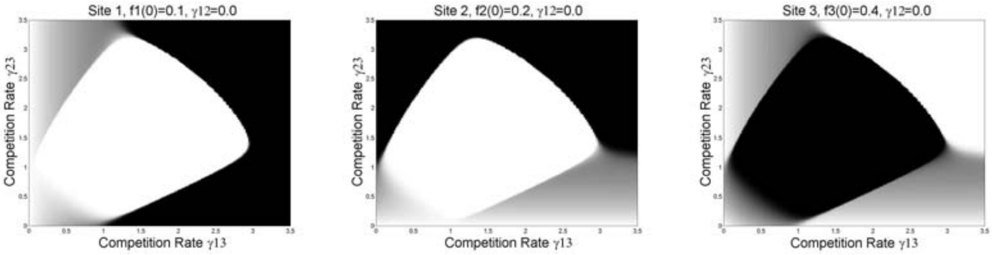

Fig. 2. This figure corresponds to a market segment of three competitors with mixed competition conditions (both, collaboration and competition are present). Each picture pixel represents the fraction of the market, f_i, controlled by each site. White means 1, black means 0. The gray color indicate a value of f_i between these two extremes. The picture on the left depicts the result of site 1, the one in the middle is associated to site 2 and the right one to site 3. The x-axis represents different values of the competition rate between sites 1 and 3. The y-axis is the one between sites 2 and 3. In all cases the competition rate between 1 and 2 is $\gamma_{12} = 0.0$. The initial conditions are $f_1(0) = 0.1$, $f_2(0) = 0.2$ and $f_3(0) = 0.4$. Observe that $f_1(0)+f_2(0) < f_3(0)$. If the initial conditions are interpreted as the initial marketing investments of the sites, we can conclude that the collaboration between sites drives to higher investment revenues.

The interesting fact is that the model predicts an unusual effect on the collaboration. Due to the nonlinearity of the differential equations, it is possible to find the appropriate conditions to obtain better investment revenues through the collaboration. Notice that the initial conditions of the sites are directly related to the initial marketing investment. As it can be observed in Fig. 2, within the appropriate competition conditions, two small sites controlling a low fraction of the market might beat a powerful dominant site if their degree of alliance is high and their degree of competition against the dominant site is medium. This result indicates that, when trying to get into a particular market segment, it is cheaper to create two small collaborating sites than to develop a huge single site.

4 Conclusions

We have seen that a simple model of the dynamical competition of the WWW is suitable to predict some important characteristics emerging in this type of markets. Although the complete analytical evaluation of the stability of the model is only possible for regular topologies of the interaction network, we have shown that, establishing a number of restrictions, it is possible to evaluate this stability for more diverse structures. Thanks to this, this model can be useful for the definition of strategies in a complex environment as the Internet.

Although these conclusions are remarkable, it is important to notice that there are many different aspects of the competition dynamics that the model does not contemplate. For example, the competition conditions are assumed to be fixed and cannot dynamically evaluate, it does not consider random or other external perturbations that could modify the behavior of the market population, cross marketing is not taken into account, etc. All these phenomena should be also appropriately modelled in order to obtain more suitable predictions and more accurate strategies.

In spite of these imperfections, we believe that this model is an interesting tool, which may be useful to understand better the interactions and collective behavior which arises in this kind of markets. In particular, the nonlinear effects appearing in the competition dynamics constitute an unexpected result, which fully justifies our interest in the model.

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