The Forecast of Predictability and Instability in Physical Modelling

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All numerical calculations have inherent inaccuracies, and beyond certain timescales even the best method will diverge from the true orbit. The concept of shadowing time allows the reliability of a computer-generated orbit to be characterized. This indicator is being applied to some models of galactic potentials, where areas of high and low predictability mix in a fractal-like way.

In the last century, the numerical approach to solving physical problems has gained in relevance with the increase of computational facilities. Methods derived from chaos theory and nonlinear dynamics techniques are quite useful in solving real problems where chaos is present and a strong dependence on initial conditions is a key issue.

Predictability refers to the assessment of the likely errors in a forecast, either qualitatively or quantitatively. It refers directly to the stability/instability of the true orbit, but also to the coincidence of the calculated orbit itself, or reliability.

Instability is a very well-known factor when considering a given solution to the model. In a broad sense, an orbit is unstable if it is strongly dependent on initial conditions. More precisely, an orbit is chaotic if the dynamics is bounded, has at least one positive Lyapunov exponent and the omega set is not periodic and does not consist solely of equilibrium points and connecting arcs. The larger the instability, the larger is the likehood that the real orbit will diverge from the calculated one.

Typically, the reliability time goes with the inverse of the Lyapunov exponent, or Lyapunov time. As all numerical calculations have inherent inaccuracies, beyond certain timescales even the best method will diverge from the true orbit. The usual steps for analysing a given system involve calculating the instability for a given set of initial conditions. It happens that the standard definition of the Lyapunov exponent has a very long convergence time (if any). Due to the slow convergence of the asymptotic value, many others numerical indexes are used, but the basic idea remains the same.

However, we can also refer to the numerical reliability of the system understood as the confidence of the computed orbit and the true orbit during an interval of time, independently of its instability. Essentially, computers move from one true orbit to another nearby orbit because of rounding errors and internal floating number representations. However, the computed orbit (called pseudo-orbit) may still lead to correct predictions because of the existence of a nearby exact solution. Otherwise, the calculated orbit may be very distant.

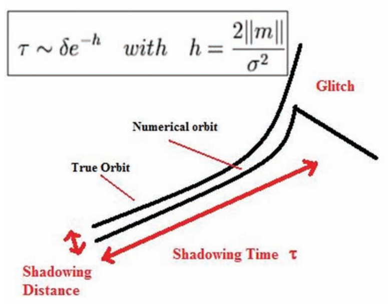

Figure 1: Computers move from one true orbit to another nearby orbit because rounding errors and internal floating number representations. How long the true orbit is shadowed by the computed orbit before suffering a glitch and later deviation can be estimated from Lyapunov exponent distributions.

The true orbit is called a shadow, and the existence of shadow orbits is a very strong property, with the shadowing time being a valid limit for the predictability of the system. A basic requirement for shadowing is that the system be hyperbolic. In case of nonhyperbolicity, the point may not be shadowed and the computed orbit behaviour may be completely different from the true one.

Stability is not the same as hyperbolicity. An orbit can be unstable and yet hyperbolic. When the orbit is nonhyperbolic, there can be still a moderate obstacle to shadowing when the nonhyperbolicity is due to tangencies between stable and unstable manifolds. But in the so-called pseudo-deterministic systems, when nonhyperbolicity is sourced to a phenomenon named unstable dimension variability, the shadowing is

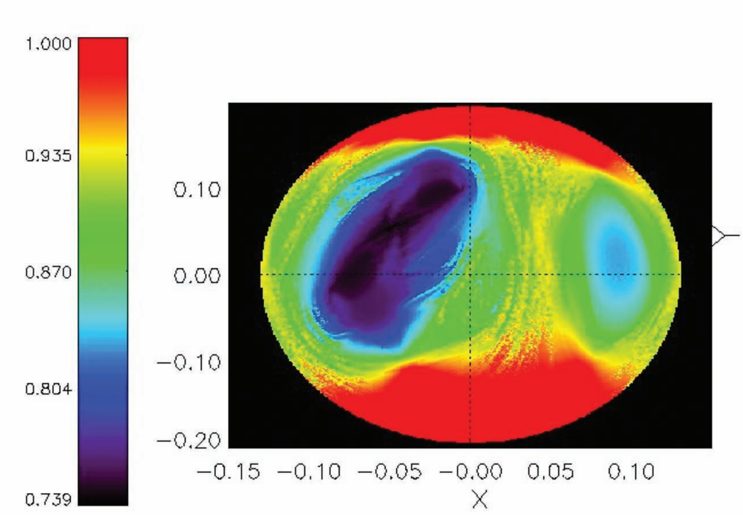

Figure 2: Plotting the instability and hyperbolicity indexes provides insight into the predictability for every initial condition of the available phase space. However, such indexes distribute in a very complex way under certain conditions of the model parameters, often showing fractal structures like those in the figure.

only valid during trajectories of given lengths, which may be very short.

Our work in the Nonlinear Dynamics, Chaos and Complex Systems Group at the Universidad Rey Juan Carlos (URJC) in Madrid, is focused on analysing and plotting predictability charts for certain models. We get the shadowing times by computing the finite time Lyapunov exponent distributions. Each computational run is based on the integration of one initial condition with a well-known Runge-Kutta-Fehlberg integrator scheme. As the integration progresses, the distribution is calculated and the instability and hyperbolicity indexes are derived. This is repeated for several conditions and parameters, thus returning a complete description about the predictability of the used model.

Note that this strategy is very well suited for high-throughput (Grid) numerical schemas. A set of shell scripts feeds the integrator with the proper initial conditions and a grid engine (in our case, GridWay) and submits the process to the proper cluster in a user transparent way.

These techniques are general enough to be applied to different models with minor modifications. Our work focuses on galactic modelling, where the applied timescales are critical. Mathematically meaningful timescales cannot be physically acceptable here. A particle being shadowed during only a few crossing times (cycles) is a critical issue. Galaxies are evolutionary entities, from the point of view of both their gravitational potential and their constituents. For instance, in a Milky Way type model, the older stars will have orbited the galactic centre just sixty times before death. And the galaxy itself may evolve during this short period of time. Interpreting the dynamical system in a broad sense, given an initial condition, the stability refers to the location itself, not to the tracer particles. But considering what happens to the model in such short timescales, and for the sake of our discussion, the forecast of the numerical galactic model reliability is then of importance.

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Realistic Material Appearance Modelling

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Physically correct and realistic visual appearance rendering or analysis of material surface visual properties require complex descriptive models capable of modelling material dependence on variable illumination and viewing conditions. While recent advances in computer hardware and virtual modelling are finally allowing the view and illumination dependencies of natural surface materials to be taken into account, this occurs at the expense of an immense increase in the required number of material sample measurements. The introduction of fast compression, modelling and rendering methods for visual data measurements is therefore inevitable.

The established practice in computer vision, graphics and pattern recognition is to base inferences only on restricted information. Virtual reality applications typically use oversimplified models that cannot even remotely approximate the appearance of real scenes, meaning human observers can easily differentiate between real and virtual scenes. Fortunately, the recent swift development of computer technology has allowed models and tools that seemed theoretical only a decade ago to become feasible as a part of foreseeable future routine processes. Physically correct and accurate material appearance visualization is in high demand. It not only has a huge economic impact in visual safety simulations and virtual design in automotive industry and architecture, but also has large potential in visual scene analytical applications, including health care, security, defect detection and content-based image retrieval. However, many challenging problems still exist, such as efficient measurement of material optical properties, image compression, optimal mathematical representation, unsupervised segmentation and interpretation and many others.

We have developed several multidimensional probabilistic models based either on a set of underlying Markov random fields or probabilistic mixtures, which allow physically correct surface lossless representation and modelling, huge measurement space compression (so far unbeaten at up to 1:1 000 000), and even modelling of previously unseen surface data or their editing. These methods are parametric, so they do not require original measurements to be stored. However, such models are nontrivial and suffer from several challenging theoretical problems such as stability, parameter estimation and non-iterative synthesis, which must be circumvented.

Alternative approaches using physical reflectance models or sophisticated sampling were also investigated. Regardless of the traits of individual models, they all meet comprehensible requirements such as unlimited seamless surface image enlargement, high visual quality and compression, as well as some less obvious features like strict separation of the analytical and synthesis parts, possible parallelization and implementation in advanced graphics hardware. Unfortunately, there is no ideal universal visual surface mathematical model suitable for all applications or material types. Each of these aforementioned models have their advantages and drawbacks simultaneously, hence optimal measurement as well as modelling depends on both material and intended application, and must be automatically recognized. Surprisingly, the reliable assessment of visual quality is also a difficult task because no usable mathematical crite-