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Amplification of the LFM signal by using piecewise vibrational methods

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Abstract

We propose the piecewise re-scaled vibrational resonance (VR) method and the piecewise twice sampling VR method to amplify the weak linear frequency-modulated (LFM) signal. The system used to amplify the weak LFM signal is a typical bistable system with fractional-order deflection nonlinearity. The concrete procedures of both the piecewise re-scaled VR method and the piecewise twice sampling VR method are explained in detail. Through studying the effect of the factional-order exponent on VR, we find that the traditional bistable system is not the optimal model to improve the weak LFM signal. By investigating different parameters on the VR phenomenon, we verify the effectiveness of the two proposed methods.

Keywords

Linear frequency-modulated signal, piecewise idea, vibrational resonance

1. Introduction

The linear frequency-modulated (LFM) signal is a common kind of signal and widely used in a variety of fields, such as in radar signal processing (Kanno et al., 2010; Elgamel and Soraghan, 2011; Zhu et al., 2011; Kronauge and Rohling, 2014), ultrasonics (Han et al., 2006; Michaels et al., 2013; Song et al., 2011) and communications (Dixon, 1994), etc. Thus, it would be relevant to find methods to amplify the weak LFM signal. There are many ways to amplify a signal. Among them, the stochastic resonance (SR) (Gammaitoni et al., 1996) and the vibrational resonance (VR) (Landa and McClintock, 2000) are two important methods that can help to amplify the weak signal. In the application of the SR and VR methods, some advanced theories are developed from the classic SR and VR theories. For example, the re-scaled SR and the twice sampling SR are put forward to extract the weak high-frequency character information from the strong noisy background (Li et al., 2007; Huang et al., 2017; Liu et al., 2017; Lu et al., 2017; Qiao et al., 2017). The re-scaled VR was proposed to amplify the weak signal with an arbitrary frequency (Liu et al., 2017; Yang et al., 2017). To our knowledge, the SR can occur in the LFM signal excited system by using the fractional Fourier transformation (Lin et al., 2016; Lin et al., 2017a, 2017b). However, there is no report on the LFM signal amplification by using VR. Furthermore, comparing with the SR, the VR is much easier to control and analyze. Moreover, the fractional Fourier transformation is not a simple method. Hence, it is necessary to propose new VR technique to amplify the weak LFM signal.

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In a linear chirp case, the instantaneous frequency $f_{in}(t)$ varies linearly with time

$$f_{in}(t) = f + \gamma t \tag{1}$$

where f is the starting frequency (at time $t = 0$), and γ is the rate of frequency increase, called chirp rate. The corresponding time-domain function for a cosine LFM signal $u(t)$ is described by

$$\begin{aligned} u(t) &= A\cos[2\pi\int_0^t f_{in}(t')\mathrm{d}t' + \phi] \\ &= A\cos(\pi\gamma t^2 + 2\pi f t + \phi) \end{aligned} \tag{2}$$

where A is the amplitude of the signal and ϕ is the initial phase, respectively. Moreover, the system in which the VR phenomenon will be discussed is a bistable system with fractional-order deflection nonlinearity, whose equation is given by

$$\frac{dx(t)}{dt} = ax(t) - bx(t)\left|x(t)\right|^{\alpha-1} + u(t) + F(t) \tag{3}$$

where $a > 0$, $b > 0$ and $\alpha > 1$. Further, α can be an integer or a factional-order number. This kind of system has been investigated broadly in the engineering fields (Kwuimy and Nbendjo, 2011; Li et al., 2012; Kwuimy et al., 2015). For the special case $\alpha = 3$, the system degenerates to the traditional bistable system. In equation (3), $F(t)$ is an auxiliary high-frequency signal and $F(t) = B\cos(\Omega t)$.

The rest of the paper is organized as follows. In Section 2, the classic VR theory is briefly discussed. Moreover, we will show that the classic VR theory is invalid in processing the LFM signal. In Section 3 and Section 4, the piecewise re-scaled VR method and the piecewise twice sampling VR method are put forward respectively to amplify the weak LFM signal. We provide the main conclusions of the paper in Section 5.

2. Ineffectiveness of the classic VR theory

In the classic VR framework, the system parameters are usually of the order of magnitude 1, and the character signal which is considered as the carrier of the useful information is a slow varied signal. If the character signal is in the fast varied form, to make the VR occur, we need to find the matched parameters by the re-scaled method (Liu et al., 2017; Yang et al., 2017). Apparently, if the character signal is a LFM signal, we cannot achieve the optimal output by the classic or re-scaled VR. This is because the frequency is time modulated and we cannot find appropriate system parameters to match the signal.

If the character signal is a harmonic signal, the response amplitude calculated by the Fourier coefficients is the index to quantify the VR phenomenon (Landa and McClintock, 2000). If the character signal is an aperiodic binary signal, the cross-correlation coefficient between the output and the input character signal is the measurement of VR (Chizhevsky and Giacomelli, 2008). Here, the LFM signal is one kind of the typical aperiodic signals. Hence, we introduce the cross-correlation coefficient as the VR performance measure. The cross-correlation coefficient between the output and the input character signal is defined by

$$C_{ux} = \frac{\sum_{j=1}^{n}[u(j) - \bar{u}][x(j) - \bar{x}]}{\sqrt{\sum_{j=1}^{n}[u(j) - \bar{u}]^2 \sum_{j=1}^{n}[x(j) - \bar{x}]^2}} \tag{4}$$

where $\bar{u}$ and $\bar{x}$ are the average of the input LFM signal and the output, respectively. The VR phenomenon may occur when the cross-correlation coefficient achieves the maximal value. If both of the excitations are in periodic form, when the index (such as the response amplitude) is calculated, we should truncate the initial time series to eliminate the transient response. However, in equation (10), the frequency of the LFM signal depends on the time. In other words, the LFM signal is a transient signal. Theoretically, the output of the system cannot achieve the steady response even in a long time. Meanwhile, to keep the completeness of the LFM signal, we cannot truncate the initial time series and use the remainder to calculate C_{ux} yet. In fact, the transient effect that usually induced by the initial conditions only has a very little effect on the C_{ux} value. In general, it does not influence the response state (near resonance or far away from resonance) of the system. In the following calculation, we let $x(0) = 0$.

We give an example in Figure 1 to illustrate the failure of the classic VR method to amplify the LFM signal. We process the LFM signal for t from 0 to 10. Other simulation parameters are in the figure caption. Although both γ and f are not large, the cross-correlation coefficient cannot achieve an ideal large value, as shown in Figure 1. This shows that the VR phenomenon cannot occur or cannot achieve an optimal state. Furthermore, if we choose longer time or larger parameters of γ and f to calculate the cross-correlation coefficient, the occurrence of VR is going to be even worse. This is because the instantaneous frequency of the signal will turn much larger and larger. We cannot find appropriate fixed parameters to match the signal, especially in a long time scale. To solve this technical problem, the piecewise idea is proposed to realize the VR phenomenon and then to amplify the weak LFM signal with arbitrary signal parameters and time length. The piecewise re-scaled VR

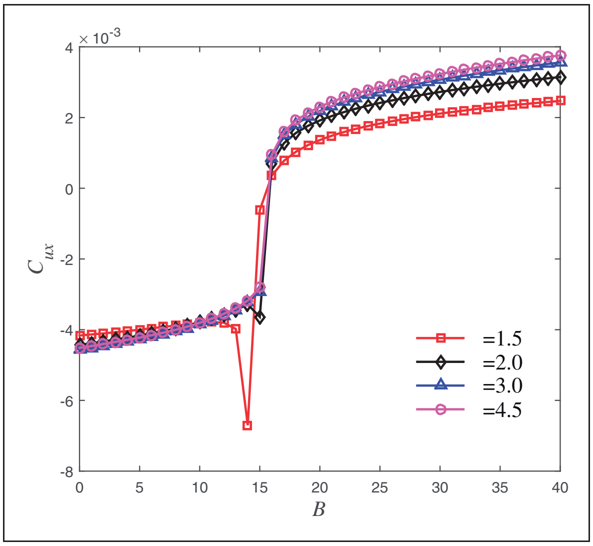

Figure 1. The cross-correlation coefficient C_{ux} versus the control parameter B under different values of α. The simulation parameters are $a = 1$, $b = 1$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$ and $\Omega = 600$.

and piecewise twice sampling VR are described in detail in the following two sections.

3. The piecewise re-scaled VR

3.1. The framework of the piecewise re-scaled VR method

At first, for a LFM signal to evolve in a long time t, we introduce the procedure of the piecewise re-scaled VR method. We divide the whole signal time series into several portions. For every segmentation, we choose different system parameters to match the signal of this subsection. We make the optimal VR phenomenon occur for every segmentation. Then, we join the output of all segmentations in turn. Finally, the whole output is compared with the original input signal to obtain the cross-correlation coefficient.

To obtain the matching condition of the parameters, we let

$$\beta = \beta_0 (f + \gamma t_{ei}) \tag{5}$$

and

$$\Omega = \Omega_0 (f + \gamma t_{ei}) \tag{6}$$

where β is the scale parameter that will be used in each segmentation and β_0 is a constant, and t_{ei} is the end time of the ith segmentation. In equation (6), Ω_0 is a constant. Hence, the frequency of the auxiliary high-frequency signal used in each segmentation is $\frac{\Omega_0}{2\pi}$ times of the maximal frequency of the LFM signal in the same segmentation.

We introduce the following time scale and coordinate transformations

$$\tau = \beta t, \quad x(t) = z(\tau) \tag{7}$$

Then, equation (3) changes to

$$\frac{dz(\tau)}{d\tau} = \frac{a}{\beta} z(\tau) - \frac{b}{\beta} z(\tau) \left| z(\tau) \right|^{\alpha - 1} + \frac{1}{\beta} u\left(\frac{\tau}{\beta}\right) + \frac{1}{\beta} F\left(\frac{\tau}{\beta}\right) \tag{8}$$

If we make

$$a_1 = \frac{a}{\beta}, \quad b_1 = \frac{b}{\beta} \tag{9}$$

then we obtain

$$\frac{dz(\tau)}{d\tau} = a_1 z(\tau) - b_1 z(\tau) \left| z(\tau) \right|^{\alpha - 1} + \frac{1}{\beta} u\left(\frac{\tau}{\beta}\right) + \frac{1}{\beta} F\left(\frac{\tau}{\beta}\right) \tag{10}$$

Through equation (10), we find that the LFM signal of each segmentation can be transformed to a slow signal by choosing an appropriate β. Meanwhile, we obtain equation (10) aiming to get the matching condition of the system parameters with the signal parameters. We must notice that the amplitude of the signal is reduced by a factor of $\frac{1}{\beta}$ compared with it in the original system described in equation (3). In order to make the system have an equivalent dynamical property with equation (3), the signal must be recovered to the original strength. As a result, we obtain the equivalent equation with system (3)

$$\frac{dz(\tau)}{d\tau} = a_1 z(\tau) - b_1 z(\tau) \left| z(\tau) \right|^{\alpha - 1} + u\left(\frac{\tau}{\beta}\right) + F\left(\frac{\tau}{\beta}\right) \tag{11}$$

When the VR phenomenon occurs in equation (11), it also occurs in the system

$$\frac{dx(t)}{dt} = a x(t) - b x(t) \left| x(t) \right|^{\alpha - 1} + \beta u(t) + \beta F(t) \tag{12}$$

In equation (11), a_1 and b_1 are small parameters and the character signal varies slowly. However, in equation (12), a and b are large parameters and the character signal can be in the fast variable form. The VR phenomenon will be investigated according to equation (12). In the following analysis, we choose the total time $t = 10$ and the whole time series of the LFM signal is divided equally into five segmentations.

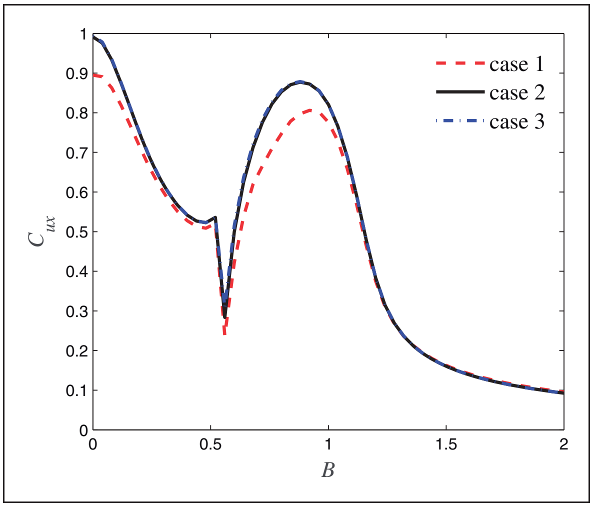

Figure 2. The cross-correlation coefficient C_{ux} versus the control parameter B under different choice of the moment t_{ei}. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.2$, $\gamma = 100$, $f = 1$, $\phi = 0$, $\beta_0 = 500$ and $\Omega_0 = 500$.

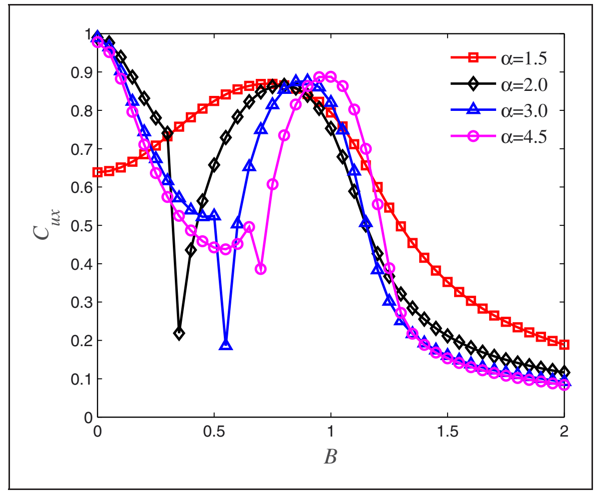

Figure 3. The cross-correlation coefficient C_{ux} versus the control parameter B under different values of α. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$, $\beta_0 = 500$ and $\Omega_0 = 500$.

The scale parameter β depends on the moment t_{ei} according to equation (5). We provide three examples to explain how to choose t_{ei}. As shown in Figure 2, the cross-correlation coefficient curve corresponds to *case 1* (t_{ei} at the initial point of the segmentation) is lower than the curves obtained by *case 2* (t_{ei} at the midpoint point of the segmentation) and *case 3* (t_{ei} at the endpoint of the segmentation). The curves obtained by *case 2* and *case 3* are almost identical. In other words, the moment t_{ei} at the midpoint or the endpoint is better than that at the initial point of the segmentation. Hence, in the following analysis, we always choose t_{ei} as the moment at the endpoint of the segmentation.

In Figure 1, we found that the classic VR method cannot amplify the LFM signal. Here, we make the VR appear for the same signal by the piecewise re-scaled VR method, as shown in Figure 3. Comparing Figure 3 with Figure 1, the maximal cross-correlation coefficient is amplified to a great extent. Specifically, at the resonance peak, the weak LFM signal is amplified significantly. Another fact, the fractional-order α has a slight effect on the resonance peak. With the increase of α, we need a larger B to induce the optimal aperiodic VR.

To show the piecewise re-scaled VR method better, we plot in Figure 4 a time series of the output that corresponds to the optimal resonance state in Figure 3. Apparently, with the evolution of t, the output is always presenting a resonance. The LFM signal is amplified greatly at any time. Once again, it illustrates the validity of the piecewise re-scaled VR method.

When using the piecewise re-scaled VR method, the parameter β_0 is a key factor that influences the scale parameter β directly. In Figure 5, the contour plot of the cross-correlation coefficient is plotted in a two-dimensional plane. Its dependence on the considered parameters is clearly shown. The resonance region is obvious in this figure. Specifically, on one hand, for a fixed parameter β_0, the variation of the other parameter B induces the VR phenomenon. On the other hand, for a fixed parameter B, the variation of β_0 can also induce the VR phenomenon. Usually, we fix the parameter β_0 in a certain signal segmentation and adjust the auxiliary high-frequency signal parameter. Then the VR phenomenon is induced.

To make the effect of the parameter β_0 on the VR phenomenon further, we provide Figure 6, which shows the curves of $C_{ux} - B$ for different values of β_0. It shows that with the increase of β_0, the location of the resonance peak moves to the left. In other words, for a larger value of β_0, the resonance occurs at smaller values of B and the corresponding peak value is larger. Furthermore, a larger peak value may indicate a stronger VR phenomenon.

3.2. Effects of the fractional-order exponent

The cross-correlation coefficient as a function of the fractional-order exponent is plotted in Figure 7 to reveal the effect of the fractional-order exponent explicitly. In this figure, for different values of B, all resonance peaks appear at $\alpha < 2$. Importantly, this fact illustrates that the traditional bistable system (for the case $\alpha = 3$) is not the optimal system for the fixed signal system. In other words, if the signals cannot be changed, we also can make the response to achieve the optimal resonance state by choosing an appropriate fractional-order exponent.

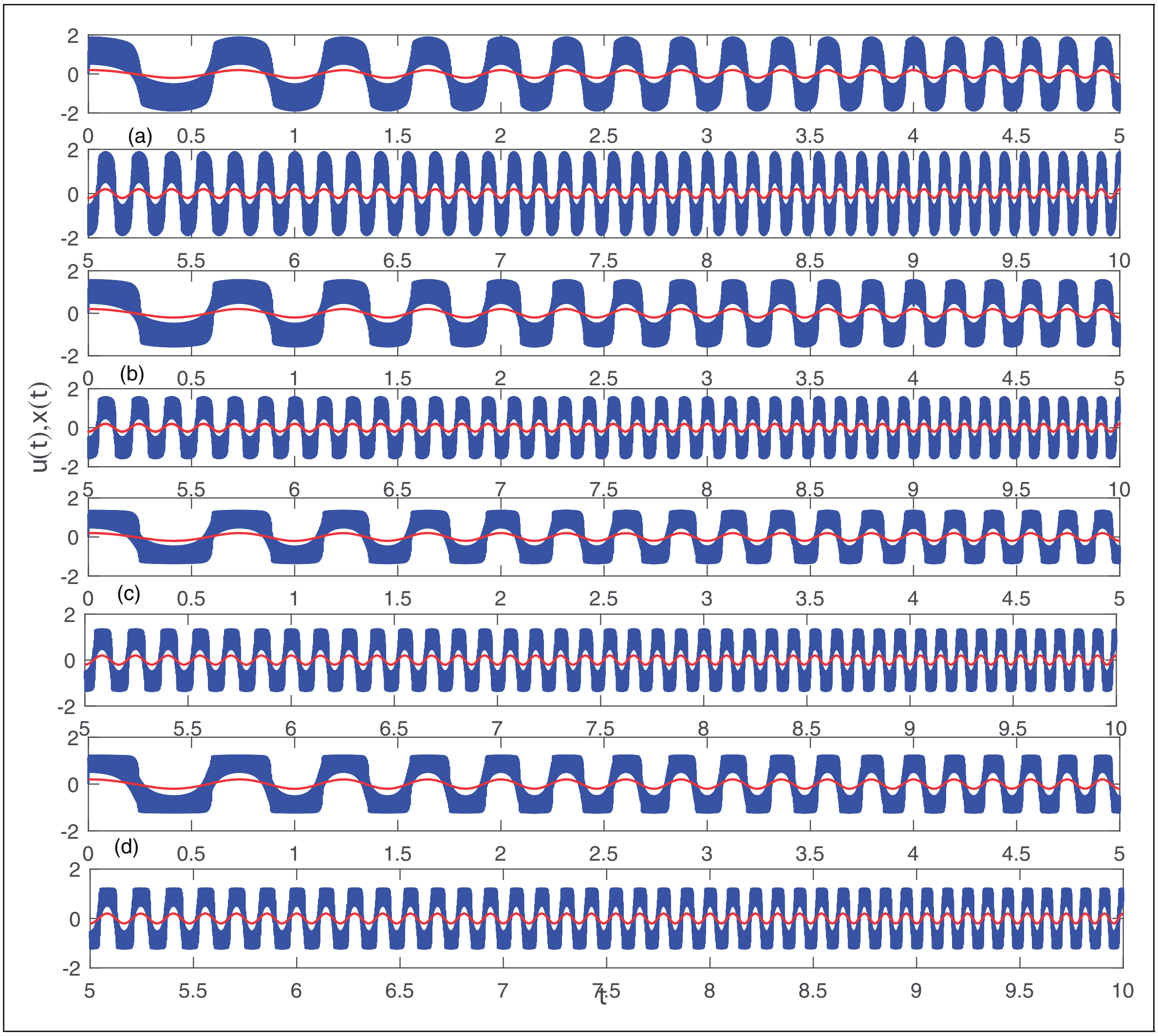

Figure 4. The weak input LFM signal and the time series of the optimal output (a) $\alpha = 1.5$, $B = 0.8$, (b) $\alpha = 2.0$, $B = 0.8$, (c) $\alpha = 3.0$, $B = 0.9$, (d) $\alpha = 4.5$, $B = 1.0$. Other simulation parameters are $a_1 = 1$, $b_1 = 1$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$, $\beta_0 = 500$ and $\Omega_0 = 500$.

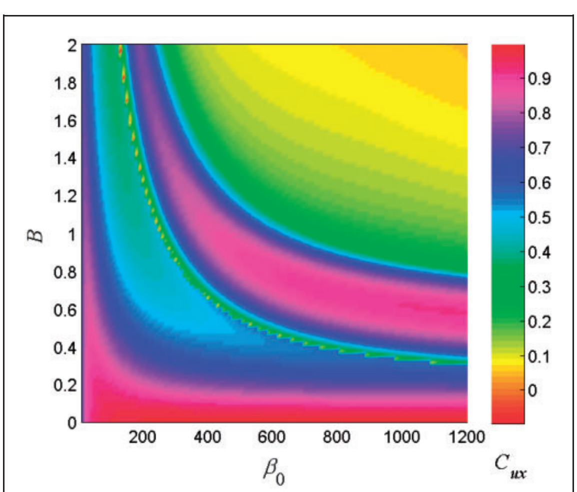

Figure 5. Contour plot of the cross-correlation coefficient in the $\beta_0 - B$ plane. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$ and $\Omega_0 = 500$.

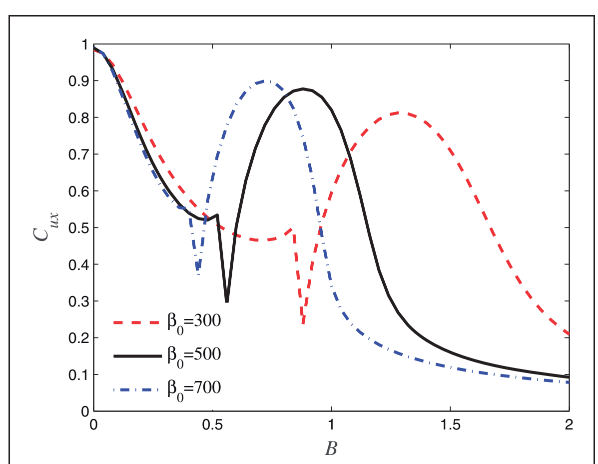

Figure 6. The cross-correlation coefficient C_{ux} versus the control parameter B under different values of β_0. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$ and $\Omega_0 = 500$.

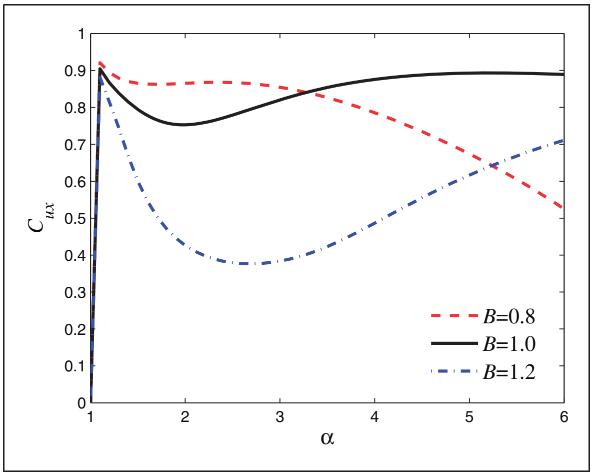

Figure 7. The cross-correlation coefficient C_{ux} versus the fractional-order exponent α under different values of B. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$, $\beta_0 = 500$ and $\Omega_0 = 500$.

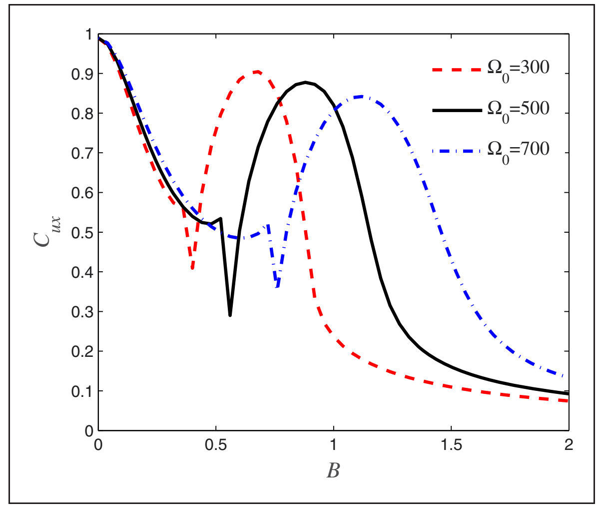

Figure 9. The cross-correlation coefficient C_{ux} versus the control parameter B under different values of Ω_0. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$ and $\beta_0 = 500$.

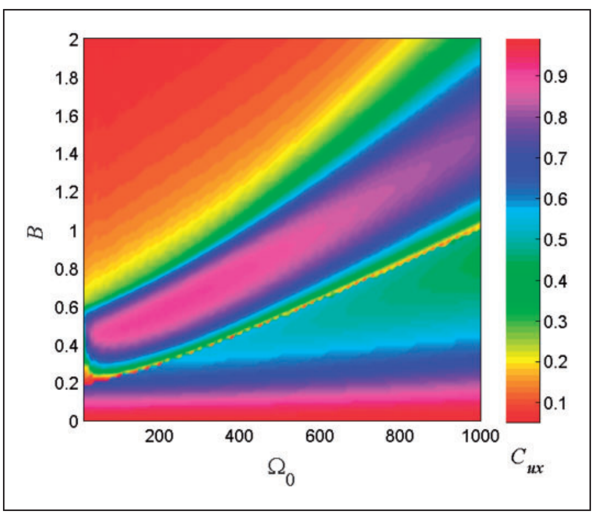

Figure 8. Contour plot of the cross-correlation coefficient in the $\Omega_0 - B$ plane. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$ and $\beta_0 = 500$.

3.3. Effects of the auxiliary high-frequency signal parameters

Besides the scale parameter and the fractional-order exponent, the frequency of the auxiliary signal is another important factor to influence VR. The contour plot of the cross-correlation coefficient is plotted in the $\Omega_0 - B$ plane in Figure 8. The value of the cross-correlated coefficient is clearly shown. The resonance region is also shown in this figure. The VR phenomenon can occur by adjusting both the amplitude or the frequency of the auxiliary signal. Once again, this illustrates the effectiveness of the piecewise re-scaled VR method.

From equation (6), we know that the high-frequency of the auxiliary signal is determined by the parameter Ω_0. To illustrate the effect of Ω further, Figure 9 shows some facts of the $C_{ux} - \Omega_0$ curves. Specifically, with the increase of Ω_0, the value of the resonance peak turns smaller and the critical value of Ω_0 which induces the strongest resonance turns larger. Moreover, there is another important fact in Figures 8 and 9: the high frequency of the auxiliary signal is far larger than the maximal frequency of the LFM signal in a certain segmentation. Based on the classic VR theory, the frequency of the auxiliary signal is about 10 times the frequency of the character signal. However, in Figures 8 and 9, the frequency of the auxiliary signal is $\frac{500}{2\pi}$ times of the maximal frequency of a certain LFM signal segmentations.

3.4. Effects of the LFM signal parameters

In this subsection, we briefly investigate the effects of the LFM signal parameters on the VR phenomenon.

Figure 10(a) shows the cross-correlation coefficient for different initial, which indicates that the initial phase does not influence the cross-correlation coefficient at all. Under different values of ϕ, the curves of $C_{ux} - B$ are completely identical. The effect of the chirp rate on VR is studied in Figure 10(b). Apparently, we can obtain the same result under different values of γ. It also reveals once again the effectiveness of the piecewise re-scaled VR method. The effect of the starting frequency on the VR is studied in Figure 10(c).

It illustrates that the starting frequency does not influence the $C_{ux} - B$ curve yet. From Figure 10, we know that the parameters of the LFM signal do not influence the effectiveness of the piecewise re-scaled VR method. It is independent of the parameters of the LFM signal.

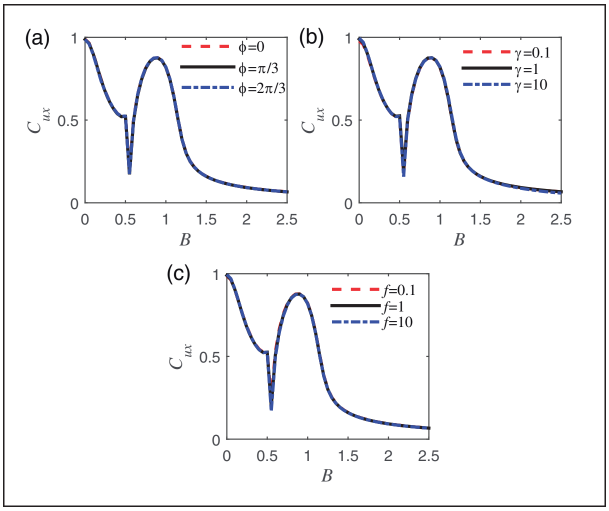

Figure 10. (a) The cross-correlation coefficient C_{ux} versus the control parameter B, in (a) $\gamma = 1$, $f = 1$, in (b) $f = 1$, $\phi = 0$, in (c) $\gamma = 1$, $\phi = 0$. Other simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.2$, $\beta_0 = 500$ and $\Omega_0 = 500$.

To verify the piecewise re-scaled VR method further, different signal parameters are adopted in Figure 11. Under these simulation parameters, the LFM signal is considered as an excitation in a high frame rate ultrasonic imaging system (Han et al., 2006). In this figure, the effectiveness of the piecewise re-scaled VR method is verified once again.

In Figure 12, we use different system parameters to carry out the simulation. Although the location and peak value of the resonance peak are related to the system parameters, the resonance is still strong and indicates the validity in the LFM signal amplification.

4. The piecewise twice sampling VR

Besides the re-scaled method, the twice sampling method is another way to process the high-frequency signal. It was first proposed by Leng et al. to extract the weak periodic character signal in a noisy background (Leng et al., 2007; Li et al., 2007). In this section, we introduce the piecewise twice sampling VR to amplify the LFM signal.

The specific implementing procedure of the piecewise twice sampling VR is as follows. First, we carry out the

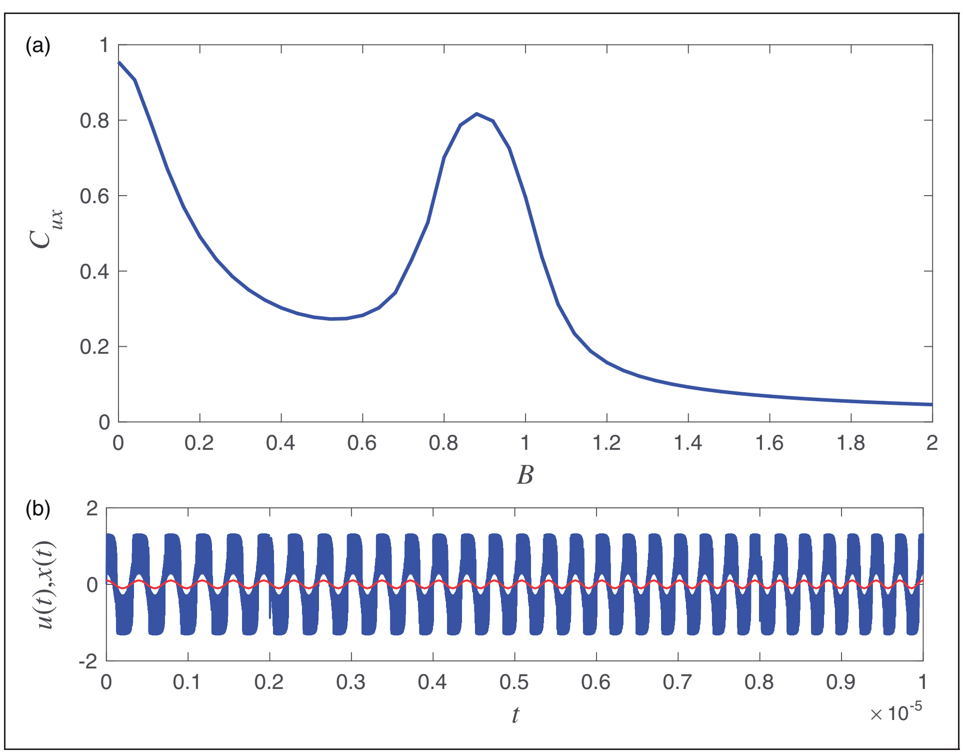

Figure 11. (a) The cross-correlation coefficient C_{ux} versus the control parameter B. (b) The weak input LFM signal and the time series of the optimal output, $B = 0.88$. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.1$, $f = 2.5 \times 10^6$, $\gamma = 1.0 \times 10^{11}$, $\phi = 0$, $\beta_0 = 500$ and $\Omega_0 = 500$.

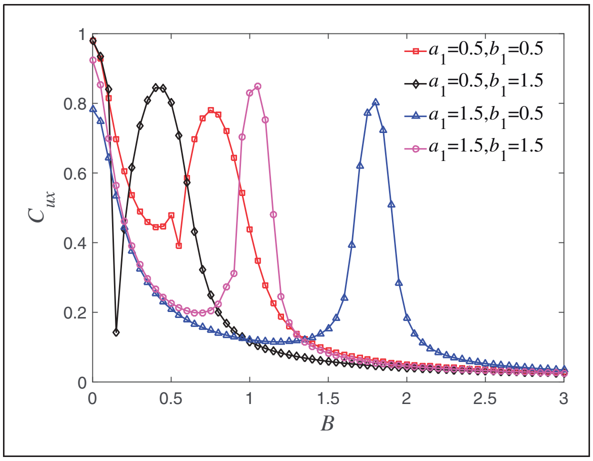

Figure 12. The cross-correlation coefficient C_{ux} versus the control parameter B. The simulation parameters are $a_1 = 1$, $b_1 = 1$, $\alpha = 3$, $A = 0.1$, $f = 2.5 \times 10^6$, $\gamma = 1.0 \times 10^{11}$, $\phi = 0$, $\beta_0 = 500$ and $\Omega_0 = 500$.

twice sampling for the original input signal. We give a constant k as the sampling transform ratio. The original sampling frequency is f_{s0}. The twice sampling frequency is f_s. Then, the sampling transform ratio $k = \frac{f_{s0}}{f_s}$. We introduce the piecewise idea to choose a different sampling transform ratio in a different LFM signal segmentation. We let $k_i = (f + \gamma t_{ei})k_0$, where k_i is the sampling transform ratio in the ith LFM signal segmentation. k_0 is a constant. t_{ei} is a time point in a certain signal segmentation which can be chosen as discussed in Section 3.1. In this section, the end point of the considered LFM signal segment is still used as the point t_{ei}. Through twice sampling of the original input signals (containing the character signal and the auxiliary signal), we obtain a new input signal. Then, we let the new signal act as the new input for the nonlinear system. If the transform ratio is an appropriate value, we can cause the aperiodic VR to occur. Finally, we reconstruct the output to the original sampling

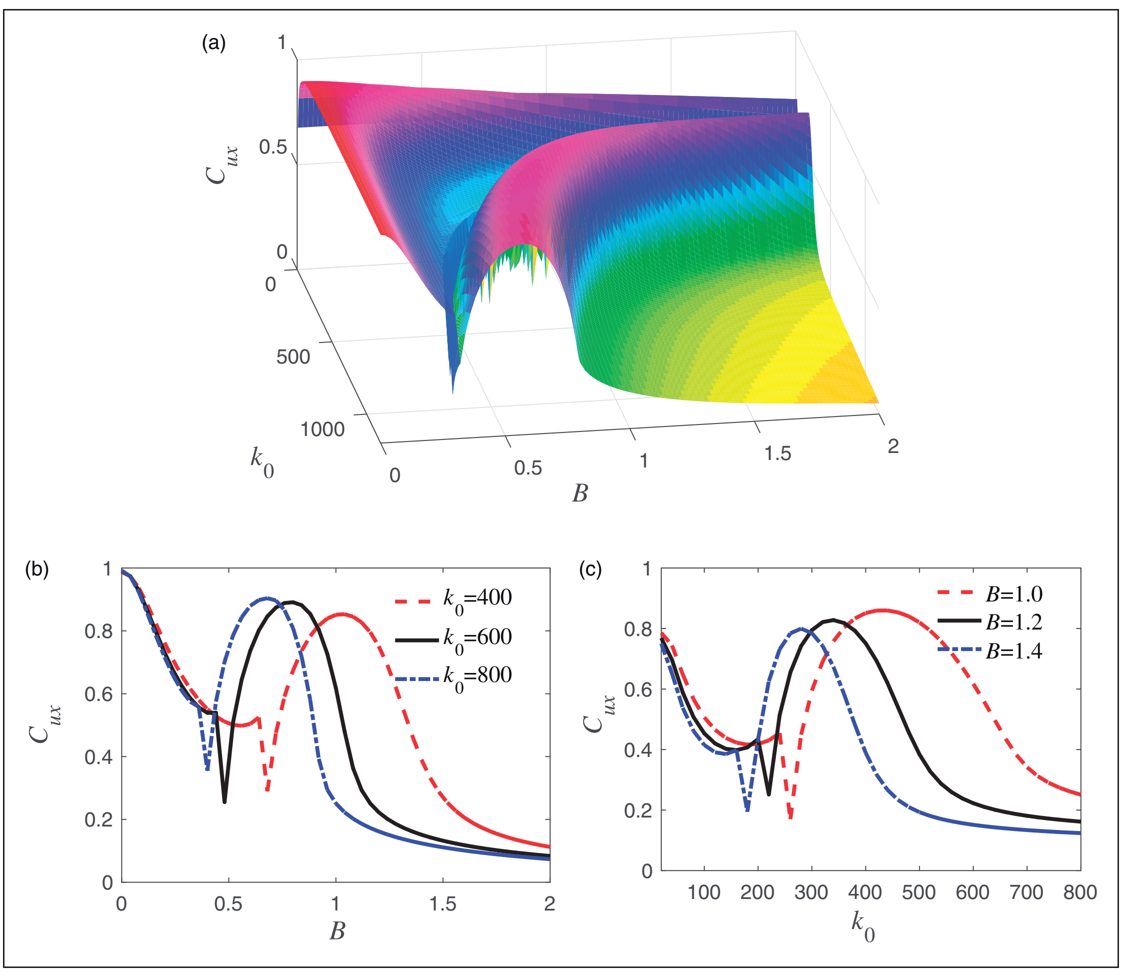

Figure 13. (a) Three-dimensional curve of C_{ux} versus B and k_0. (b) Two-dimensional plot of C_{ux} versus B under different values of k_0. (c) Two-dimensional plot of C_{ux} versus k_0 under different values of B. The simulation parameters are $a = 1$, $b = 1$, $\alpha = 3$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$ and $\Omega_0 = 500$.

according to the sampling transform ratio. The new output is compared with the original signal to calculate the cross-correlation coefficient. At the peak of the cross-correlation coefficient curve, the resonance may occur showing that the weak aperiodic signal has been amplified. Some examples are given to verify the effectiveness of the piecewise twice sampling VR method in the following.

The constant k_0 is a key parameter to determine the sampling transform ratio. Then, it determines the maximal frequency in each signal segmentation. In Figure 13, there are obvious resonance regions presented. The VR can be realized by adjusting B or k_0. For adjusting B, the resonance is realized in the traditional VR mechanism. For adjusting k_0, the resonance is achieved at an optimal value. It is because k_0 reduces the temporary frequency of the LFM signal. If the system is excited by a harmonic signal, based on the famous frequency response curve, the response amplitude of the system will have the maximal value only at an optimal value. Neither a too small or too large value of the excitation frequency can induce strong response. Similarly, if the maximal frequency of the LFM signal is too large or too small, it cannot induce a strong response yet. Moreover, the maximal frequency of the LFM signal depends on k_0 monotonically. In other words, with the increase of k_0, the reduced frequency of the LFM signal will decrease gradually. Only an optimal value of the maximal frequency of the LFM signal can make the response reach the strongest resonance. As a result, the curve of C_{ux} versus k_0 will also present the resonance phenomenon. Figure 13 shows the feasibility of the piecewise twice sampling VR method.

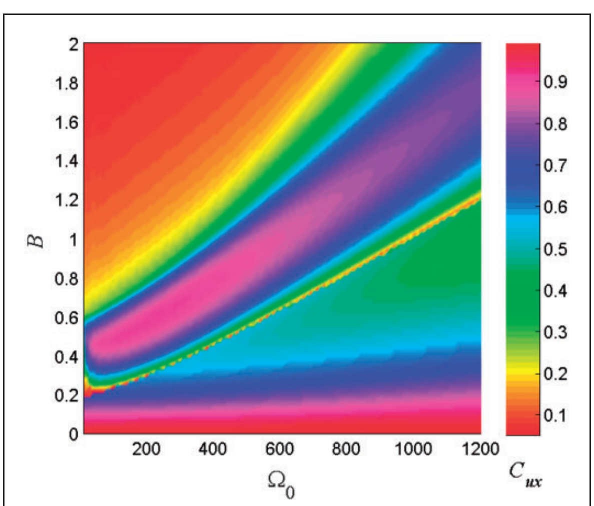

Figure 14. Contour plot of the cross-correlation coefficient in the $\Omega_0 - B$ plane. The simulation parameters are $a = 1$, $b = 1$, $\alpha = 3$, $A = 0.2$, $\gamma = 1$, $f = 1$, $\phi = 0$ and $k_0 = 500$.

The dependence of the cross-correlation coefficient on the auxiliary signal parameters B and Ω_0 is clearly shown in Figure 14. There is a resonance region appearing in the plot. In other words, the VR occurs in this region and the weak signal can be excellently amplified. Both the parameter B and the parameter Ω_0 can induce the resonance. By the piecewise twice sampling VR, the LFM signal can be amplified to a great degree.

5. Conclusions

The LFM signal is widely used in engineering. Due to its frequency in modulated form, it is difficult to amplify it by the classic VR method. In view of this problem, we introduce the piecewise idea in the VR theory to process the LFM signal. Specifically, the piecewise re-scaled VR method and the piecewise twice sampling VR method are proposed to amplify the weak LFM signal.

The bistable system is commonly used in studying the VR phenomenon or the similar SR phenomenon. In this paper, we use a general bistable system which has a nonlinearity with fractional-order exponent. If the excitations are fixed, we can make VR occur by adjusting the fractional-order exponent of the nonlinearity. Most of the time, when the optimal VR occurs, the traditional bistable system may not be the best nonlinear model to amplify the weak LFM signal.

For the piecewise re-scaled VR method, the scale parameter used in different signal segmentations is the key parameter. For the piecewise twice sampling method, the frequency transform ratio used in different signal segmentations is the key parameter. By choosing an appropriate scale parameter or frequency transform ratio VR can occur easily and optimally. We show that both methods are effective in amplifying the weak LFM signal by showing different examples. Further, the two methods are independent of the parameters of the LFM signal, what indicates the generality of both methods.

By comparing two new VR methods with the classical VR method, we see that there are two major differences. The first one is the piecewise idea in processing the frequency modulated signal. The second one is the choice of the frequency of the auxiliary signal. In the classical VR theory framework, the frequency of the auxiliary signal is usually about ten times the harmonic character signal. However, if the character signal is in the LFM signal form, the frequency of the auxiliary signal is about a hundred times the maximal frequency of a certain LFM signal segmentation.

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